SETTING PRESCRIBING BUDGETS IN GENERAL PRACTICE: A BAYESIAN RESOURCE ALLOCATION MODEL

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Abstract

Each general practice in England is required to have a prescribing budget, but there is continuing debate about what is the best way to distribute Health Authority/Primary Care Group funds between practices. Since capitation alone fails to account for all relevant practice-level variables, a mixture of capitation and bilateral negotiation has been used to try to incorporate as many variables as possible in the process. However, this is cumbersome and costly, susceptible to strategic behaviour by practices, and difficult to scrutinise. This paper develops a Bayesian Resource Allocation Model (BRAM) which directly computes 'optimal' budget allocations for individual practices. Unlike any existing capitation formula, the BRAM can incorporate all relevant data, including historical costs and ASTRO-PUs. By explicitly formulating all uncertainties in terms of probability distributions, the BRAM also makes the allocation process more transparent than the 'capitation plus negotiation' approach. The BRAM's empirical performance is assessed using 1993-94 data on 96 Lincolnshire practices.

Key words: prescribing, budgets, Bayesian, resource allocation

JEL Classification: C11, C51, H51, Il0

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1. Introduction

The need for a transparent and rational basis on which to set practice-level prescribing budgets has been made more urgent by the Government White Paper *The New NHS: Modern, Dependable* [1], which led to the creation of 500 Primary Care Groups (PCGs) throughout England in April 1999. Each Health Authority (HA) has around 5 PCGs in its area, and retains overall responsibility for balancing the combined budgets of all its health care commisioners. In turn, each PCG incorporates around 20 practices, and has an overall budget for the purchase of hospital and community health services, prescribing, and general practice infrastructure. Finally, each individual practice in a PCG has an 'indicative' budget for the full range of services. The White Paper explicitly requires each individual practice to have a prescribing budget [2], but there is continuing controversy about how HA/PCG prescribing funds should be allocated to practices, and prescribing costs continue to rise rapidly. They now account for well over a tenth of all National Health Service spending [3]. This paper considers the problem of allocating prescribing funds to individual practices from a HA perspective, but similar considerations apply to PCGs.

Many HAs have considered introducing capitation-based prescribing budgets for their general practices in response to advice from the National Health Service Executive (NHSE). Guidance from the NHSE suggests that previous years' expenditures be used as a starting point. From this basis, HAs are required to consider an uplift plus growth factor for practices whose budget share, adjusted for the demographics of practice lists and other need factors, is below the local average. The two key inputs of information in this process are previous years' expenditures, and data enabling adjustments for practice demographics. The latter are achieved through the use of ASTRO-PUs (Age, Sex, and Temporary Resident Originating Prescribing Units), which provide capitation weights for practice lists reflecting the prescribing costs of 18 different age-sex groups [4, 5]. However, there are wide variations in prescribing costs across HAs and practices, of which only around 25 per cent are explained by ASTRO-PUs [4]. Other correlates have been proposed, including out-of-hours services and prescription charge exemptions [6], but these explain only about 30-40 per cent of the additional variation. The rest is widely viewed as *prima facie* evidence of inefficient and/or inappropriate prescribing by general practices.

Attempts have been made to incorporate additional variables in the budgetary process through bilateral negotiations between HAs and individual general practices. However, there is concern that this can give rise to incentives for strategic behaviour among practices, and to allocations which are not related to a consistent concept of 'need'. Recent research has started to investigate the feasibility of deriving a 'needs-based formula' for distributing prescribing funds at practice level in order to promote equitable access to health care. So far, this research has focused primarily on the problem of identifying the remaining factors responsible for variations in prescribing costs, over and above those explained by ASTRO-PUs [7].

A fundamental question remains as to how the different types of information (previous years' expenditures, ASTRO-PUs, and any other relevant variables) should actually be used to set practice-level prescribing budgets. The main objectives of the budget setting process are to meet ‘need’, and to divide the HA’s budget ‘fairly’ between its general practices but, to the authors’ knowledge, there is currently no single formula enabling all the relevant data to be combined in a transparent and rational way in order to compute ‘fair’ budget allocations. Setting budgets by amending historical costs using ASTRO-PUs and other information in bilateral negotiations is a costly and cumbersome process that could still result in inequitable allocations. There is nothing to ensure that the available information is used appropriately and consistently from one case to the next, and the procedure is not ‘transparent’ to general practices or the public.

One possible solution is to rely on capitation formulas alone, using ASTRO-PUs (or some other weighting system) to compute the weighted sum in the denominator. Capitation formulas are simple, transparent, and lead to allocations which are ‘fair’ in an ‘equal share’ sense, since they give an equal share of the HA’s budget per ASTRO-PU to each practice. However, the conventional view of ‘fairness’ is that money should be allocated to each practice according to the ‘need’ (defined as ‘capacity to benefit’) of the patients registered with it, so that practices whose patients have an equal capacity to benefit from available interventions should be set equal budgets per head [8]. This is an ‘end-state’ conceptualisation of fairness (i.e. it is concerned with where resources actually end up) that is unlikely to be achieved by pure capitation methods, since weighted units such as ASTRO-PUs explain so little of the variation in prescribing costs across practices. It is likely that at least some of the unexplained variation is due to differences in the clinical characteristics of practice populations, or to the fact that some general practices are better at identifying and treating groups of patients (such as asthmatics or heart disease patients) who require high cost and/or long-term drug treatments [3]. Pure capitation methods may thus lead to unfair reductions in some practices’ budgets, and excessively large increases in the budgets of others.

The capitation mechanism is also unduly wasteful of data. It does not make use of any of the potentially valuable information contained in routinely-collected historical cost data, such as previous year’s Net Ingredient Cost. There is a danger that naively incorporating historical costs in the allocation mechanism might tend to institutionalise inappropriate prescribing behaviour. However, historical costs might also reflect legitimate variables (e.g. epidemiological factors) that are difficult to measure directly for each practice, and this information should be included in the allocation process. Recent research confirms that historical costs are systematically related to ‘need’ factors [7]. An attempt could be made to extend the range of variables encapsulated in the capitation mechanism by increasing the number of categories in the weighting system. However, the ASTRO-PU system already has 18 categories (Table 1), and this number would quickly rise to impracticable proportions as more variables were incorporated.

Given the informational problems involved in measuring ‘need’, other concepts of ‘fairness’ may be more relevant to the comparison of alternative allocation models for prescribing funds. The most obvious is a ‘fair process’ conceptualisation, in which allocations are made on the basis of rules that are agreed to be ‘fair’ by participants. Note that capitation methods, whether based on ASTRO-PUs or other weighting systems, will fail to be fair in the ‘fair process’ sense if they ignore information on variables which practices perceive as being relevant to their claim on the HA's total prescribing budget. A potentially fruitful 'fair process' approach explored in this paper is to use an *econometric framework* attempting to encompass all relevant data. This paper develops a Bayesian Resource Allocation Model (BRAM) which enables 'optimal' budget allocations to be directly computed for each practice. Unlike any existing capitation formula, the model can incorporate all currently available information, including historical costs, weightings for age, sex and chronic illness, and any other practice-level variables considered relevant. It estimates coefficients for these variables, indicating their contribution to the optimal budget allocations, and standard errors for all estimated coefficients and budgets. The allocations are 'optimal' in the sense that they minimise an expected quadratic loss function, given the available information.

The BRAM does not solve the problem of what variables *should* be included in the budget allocation process, but it offers a new approach to combining the different types of information. By explicitly formulating all uncertainties in terms of probability distributions, the Bayesian approach forces budget setters to be explicit about the relative weight attached to each type of information, making it more transparent to general practices and the public than the 'capitation plus negotiation' approach. It is argued that the BRAM is 'fair' in a 'fair process' sense, while encompassing population 'need' as far as is possible given current knowledge, since it can incorporate not only ASTRO-PUs, but all other relevant practice-level data. The model's performance is explored in an empirical application using 1993-94 data on 96 Lincolnshire practices. It is found that a simple specification of the BRAM, based on Net Ingredient Cost and ASTRO-PUs in 1993-94, can almost replicate the 96 actual budget allocations for 1994-95. This suggests that the process of 'capitation plus negotiation' can realistically be superseded by the Bayesian approach.

Section 2 briefly reviews the Bayesian approach to statistical analysis on which the BRAM is based. The BRAM is then set out in full in Section 3. Section 4 describes the data used in the empirical application, and Section 5 reports the results. Section 6 concludes the paper.

2. The Bayesian approach to inference

The Bayesian approach is particularly suitable for the problem of estimating practice-level prescribing budgets within a given HA because, unlike classical maximum likelihood methods, it does not rely on asymptotic theory. The latter is a 'frequentist' theory that is justified in terms of arbitrarily large numbers of observations, but the number of practices within a given HA is fixed, and always relatively small (of the order of 100). In addition, the problem involves estimating one parameter (the 'optimal' budget) for each practice, so each additional observation raises the number of parameters to be estimated by one. These cannot be estimated consistently using ordinary (i.e. classical) methods, because the number of parameters increases without limit as the sample size goes to infinity [9]. Since asymptotic theory is not relevant to Bayesian analysis, however, these parameters present no problem in the BRAM.

To clarify the general procedure followed in setting up the BRAM, this section briefly reviews the Bayesian approach to statistical analysis [10, 11]. Let x and y denote two random variables, and h(x, y) their joint probability density function (p.d.f.). This joint p.d.f. can be written in terms of the conditional and marginal p.d.f.s of x and y as

$$h(x, y) = g(x|y)\cdot f(y) = f(y|x)\cdot g(x)$$

(1)

where g(x|y) and f(y|x) are the conditional p.d.f.s, and g(x) and f(y) are the corresponding marginal p.d.f.s. A simple rearrangement of formula (1) gives the basic form of Bayes' Theorem on which this paper is based:

$$g(x|y) = f(y|x)\cdot g(x)/f(y)$$

(2)

Suppose a parameter vector $\boldsymbol{\theta}$ is to be estimated. The Bayesian approach treats $\boldsymbol{\theta}$ as a random variable, and **y** as a given data vector. Bayes' Theorem above then becomes

$$g(\boldsymbol{\theta}|\mathbf{y}) = f(\mathbf{y}|\boldsymbol{\theta}) \times g(\boldsymbol{\theta})/f(\mathbf{y})$$

(3)

where $g(\boldsymbol{\theta}|\mathbf{y})$ is the 'posterior' p.d.f. of $\boldsymbol{\theta}$, $f(\mathbf{y}|\boldsymbol{\theta})$ is interpreted as a conventional likelihood function, $g(\boldsymbol{\theta})$ is the 'prior' p.d.f. of $\boldsymbol{\theta}$, and $f(\mathbf{y})$ is treated as a constant, since it does not involve $\boldsymbol{\theta}$. It is conventional to absorb all constants into the 'proportionality constant' of $g(\boldsymbol{\theta}|\mathbf{y})$, and to write (3) as

$$g(\boldsymbol{\theta}|\mathbf{y}) \propto f(\mathbf{y}|\boldsymbol{\theta}) \times g(\boldsymbol{\theta})$$

(4)

where the symbol $\propto$ means 'is proportional to'.

Formula (4) provides a systematic and consistent way to update information about the distribution of the parameters. Given an appropriate prior distribution, and new information encapsulated in a likelihood function, the parameters' posterior distribution is obtained by multiplying the likelihood by their prior distribution.

Note that the Bayesian procedure gives the whole posterior distribution, not a point estimate. To obtain a point estimate, a suitable loss function (e.g. quadratic) can be chosen. The 'optimal' estimate is then the value of $\boldsymbol{\theta}$ that minimises the expected loss, where the expectation is taken with respect to the posterior p.d.f. of $\boldsymbol{\theta}$. With a quadratic loss function, the optimal estimate is the mean of the posterior distribution. An alternative approach is to choose the value of $\boldsymbol{\theta}$ that maximises the posterior p.d.f. $g(\boldsymbol{\theta}|\mathbf{y})$. This is known as 'generalised maximum likelihood' estimation, and yields the same result as quadratic loss minimisation when $g(\boldsymbol{\theta}|\mathbf{y})$ is unimodal and symmetric (e.g. a multivariate normal or t-distribution).

3. The Bayesian Resource Allocation Model (BRAM)

The modelling procedure begins by specifying econometric models for the prior p.d.f. and the likelihood function involving the parameters of interest (namely, the practice budgets, and the coefficients of ASTRO-PUs and of any additional variables in the model). The prior p.d.f., likelihood, and posterior p.d.f. are then calculated, and used to find 'optimal' estimators of the parameters. Each of these stages is now described.

3.1. Econometric models for the prior p.d.f. and the likelihood function

Assume that there are N individual practices under the auspices of a given HA. Let B_i^* ≡ practice i's 'optimal' budget (e.g. the one that most closely reflects 'need'), HCB_i ≡

practice i's historical-cost-based budget (e.g. historical cost plus an uplift). The prior is based on the linear model

$$\mathbf{b}^* = \mathbf{hcb} + \boldsymbol{\varepsilon}_1 \tag{5}$$

where $\mathbf{b}^* \equiv (b_1^*, \ldots, b_N^*)'$, $b_i^* \equiv \ln B_i^*$, $\mathbf{hcb} \equiv (hcb_1, \ldots, hcb_N)'$, $hcb_i \equiv \ln HCB_i$ and $\boldsymbol{\varepsilon}_1 \equiv (\varepsilon_{11}, \varepsilon_{12}, \ldots, \varepsilon_{1N})'$. $\boldsymbol{\varepsilon}_1$ is a multivariate normally distributed disturbance vector i.e. $\boldsymbol{\varepsilon}_1 \sim MVN(\mathbf{0}, \sigma^2\mathbf{A}_1^{-1})$, where $\mathbf{A}_1$ is a $(N\times N)$ positive-definite symmetric matrix that is specified in advance (it is not estimated). For example, if the disturbances are believed to be independent across practices and homoskedastic, the appropriate $\mathbf{A}_1$ is the $(N\times N)$ identity matrix, denoted here by $\mathbf{I}_{N\times N}$. σ is a 'nuisance parameter' that must be accounted for in the calculations, but is not estimated.

The model represented by equation (5) expresses the prior belief that the optimal budgets this year are just the practices' historical-cost-based budgets, with a random error. Note that using *natural logarithms* in **hcb** will ensure that estimated budgets are never negative.

Let $\mathbf{X}_i \equiv (X_{i1}, X_{i2}, \ldots, X_{iK})'$ be a $(K\times 1)$ vector of characteristics pertaining to practice i, and $\boldsymbol{\beta} \equiv (\beta_1, \beta_2, \ldots, \beta_K)'$ a parameter vector common to all practices. The likelihood function is based on the linear model

$$\mathbf{b} = \mathbf{b}^* + \boldsymbol{\varepsilon}_2$$
$$\mathbf{b} = \mathbf{X}\boldsymbol{\beta} \tag{6}$$

where $$\mathbf{X} \equiv \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1K} \\ X_{21} & X_{22} & \cdots & X_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N1} & X_{N2} & \cdots & X_{NK} \end{bmatrix}$$

is a $(N\times K)$ data matrix obtained by stacking the vectors $\mathbf{X}_i'$, and $\boldsymbol{\varepsilon}_2 \sim MVN(\mathbf{0}, \sigma^2\mathbf{A}_2^{-1})$ is a disturbance vector. As before, the matrix $\mathbf{A}_2$ must be specified in advance.

The model represented by equations (6) says that a linear combination of the characteristics pertaining to each practice, using the elements of $\boldsymbol{\beta}$ as weights, gives the natural logarithm of each practice's optimal budget allocation, plus a random error. The error term in (6) can be assumed to derive from measurement errors in one or more elements of the **X** matrix (under the reasonable assumptions that these are additive and normally distributed), and/or from other random factors that cause deviations from the optimal budget allocations.

The statistical procedure developed below enables estimation of $\mathbf{b}^*$ (the optimal allocation is the antilogarithm of $\mathbf{b}^*$), the coefficient vector $\boldsymbol{\beta}$ in equations (6), and the standard errors of the estimates. Note that $\mathbf{b}^*$ is independent of **X** in the prior model represented by equation (5), and **b** is independent of **hcb** in the likelihood model represented by equations (6).

3.2. Derivation of the prior p.d.f., the likelihood, and the posterior p.d.f.

Given (5) and the independence of $\mathbf{b}^*$ and **X**, the prior p.d.f. of $\mathbf{b}^*$, given σ, **X** and **hcb**, is multivariate normal with mean **hcb** and covariance matrix $\sigma^2\mathbf{A}_1^{-1}$. The p.d.f. is

$$g(\mathbf{b}^*|\sigma, \mathbf{X}, \mathbf{hcb}) = g(\mathbf{b}^*|\sigma, \mathbf{hcb}) = (2\pi)^{-N/2}\sigma^{-N}|\mathbf{A}_1|^{1/2}\exp\{-(\mathbf{b}^*-\mathbf{hcb})'\mathbf{A}_1(\mathbf{b}^*-\mathbf{hcb})/2\sigma^2\}$$
$$\propto \sigma^{-N}\exp\{-(\mathbf{b}^*-\mathbf{hcb})'\mathbf{A}_1(\mathbf{b}^*-\mathbf{hcb})/2\sigma^2\}$$
(7)

It is assumed that there is no prior information about β or σ. This is expressed by specifying a joint 'uninformative improper' p.d.f. for β and σ of the form [10, p. 150]

$$g(\beta, \sigma) \propto 1/\sigma$$
(8)

A joint prior p.d.f. for ($\mathbf{b}^*$, β, σ) (given $\mathbf{X}$ and $\mathbf{hcb}$) is now obtained by multiplying together (7) and (8):

$$g(\mathbf{b}^*, \beta, \sigma|\mathbf{X}, \mathbf{hcb}) \propto \sigma^{-N-1}\exp\{-(\mathbf{b}^*-\mathbf{hcb})'\mathbf{A}_1(\mathbf{b}^*-\mathbf{hcb})/2\sigma^2\}$$
$$= \sigma^{-N-1}\exp\{-(\mathbf{hcb}-\mathbf{b}^*)'\mathbf{A}_1(\mathbf{hcb}-\mathbf{b}^*)/2\sigma^2\}$$
(9)

Given (6) and the independence of $\mathbf{b}$ and $\mathbf{hcb}$, the p.d.f. of $\mathbf{b}$, conditional on $\mathbf{b}^*$, β, σ and $\mathbf{hcb}$ is multivariate normal with mean $\mathbf{b}^*$ and covariance matrix $\sigma^2\mathbf{A}_2^{-1}$. This is interpreted as a likelihood function for ($\mathbf{b}^*$, β, σ), conditional on $\mathbf{X}$ and $\mathbf{hcb}$:

$$l(\mathbf{b}^*, \beta, \sigma|\mathbf{X}, \mathbf{hcb}) = l(\mathbf{b}^*, \beta, \sigma|\mathbf{X}) = (2\pi)^{-N/2}\sigma^{-N}|\mathbf{A}_2|^{1/2}\exp\{-(\mathbf{X}\beta-\mathbf{b}^*)'\mathbf{A}_2(\mathbf{X}\beta-\mathbf{b}^*)/2\sigma^2\}$$
$$\propto \sigma^{-N}\exp\{-(-\mathbf{b}^*+\mathbf{X}\beta)'\mathbf{A}_2(-\mathbf{b}^*+\mathbf{X}\beta)/2\sigma^2\}$$
(10)

Using Bayes' Theorem, the joint posterior p.d.f. for ($\mathbf{b}^*$, β, σ) (conditional on $\mathbf{X}$ and $\mathbf{hcb}$) is now obtained by multiplying the prior in (9) by the likelihood in (10):

$$g(\mathbf{b}^*, \beta, \sigma|\mathbf{X}, \mathbf{hcb})$$
$$\propto \sigma^{-2N-1}\exp\{-[(\mathbf{hcb}-\mathbf{b}^*)'\mathbf{A}_1(\mathbf{hcb}-\mathbf{b}^*)+(-\mathbf{b}^*+\mathbf{X}\beta)'\mathbf{A}_2(-\mathbf{b}^*+\mathbf{X}\beta)]/2\sigma^2\}$$
(11)

The remaining exposition is considerably simplified if the posterior p.d.f. in (11) is rewritten as

$$g(\mathbf{b}^*, \beta, \sigma|\mathbf{X}, \mathbf{hcb}) \propto \sigma^{-2N-1}\exp\{-[(\mathbf{w} - \mathbf{GZ})'(\mathbf{w} - \mathbf{GZ})]/2\sigma^2\}$$
(12)

where

$$\mathbf{w} = \begin{pmatrix} \mathbf{A}_1^{1/2}\mathbf{hcb} \\ \mathbf{0}_{N\times 1} \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} \mathbf{A}_1^{1/2} & \mathbf{0}_{N\times K} \\ \mathbf{A}_2^{1/2} & -\mathbf{A}_2^{1/2}\mathbf{X} \end{pmatrix} \quad \mathbf{Z} = \begin{pmatrix} \mathbf{b}^* \\ \beta \end{pmatrix}$$

and $\mathbf{0}_{N\times K}$ denotes an (N×K) null vector. Note that

$$\mathbf{G}'\mathbf{w} = \begin{pmatrix} \mathbf{A}_1\mathbf{hcb} \\ \mathbf{0}_{K\times 1} \end{pmatrix} \quad \text{and} \quad \mathbf{G}'\mathbf{G} = \begin{pmatrix} (\mathbf{A}_1+\mathbf{A}_2) & -\mathbf{A}_2\mathbf{X} \\ -\mathbf{X}'\mathbf{A}_2 & \mathbf{X}'\mathbf{A}_2\mathbf{X} \end{pmatrix}$$
(13)

Using a basic formula for the inverse of a partitioned matrix, it can also be shown that

$$(\mathbf{G}'\mathbf{G})^{-1} = \begin{pmatrix} (\mathbf{A}_1+\mathbf{A}_2)^{-1}(\mathbf{I}_{N\times N}+\mathbf{A}_2\mathbf{XMX}'\mathbf{A}_2(\mathbf{A}_1+\mathbf{A}_2)^{-1}) & (\mathbf{A}_1+\mathbf{A}_2)^{-1}\mathbf{A}_2\mathbf{XM} \\ \mathbf{MX}'\mathbf{A}_2(\mathbf{A}_1+\mathbf{A}_2)^{-1} & \mathbf{M} \end{pmatrix} \quad (14)$$

where

$$\mathbf{M} = (\mathbf{X}'\mathbf{A}_2\mathbf{X} - \mathbf{X}'\mathbf{A}_2(\mathbf{A}_1+\mathbf{A}_2)^{-1}\mathbf{A}_2\mathbf{X})^{-1}$$
(15)

3.3. Derivation of 'optimal' estimators from the posterior p.d.f.

To derive the optimal estimator of the parameter vector $\mathbf{Z}$, the following decomposition result is required:

$$(\mathbf{w}\text{-}\mathbf{GZ})'(\mathbf{w}\text{-}\mathbf{GZ}) = (\mathbf{Z}\text{-}\mathbf{Z}^*)'(\mathbf{G}'\mathbf{G})(\mathbf{Z}\text{-}\mathbf{Z}^*) + (\mathbf{w}\text{-}\mathbf{GZ}^*)'(\mathbf{w}\text{-}\mathbf{GZ}^*)$$

(16)

where

$$\mathbf{Z}^* \equiv (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{w}$$

(17)

Substituting (16) into (12) gives

$$g(\mathbf{b}^*, \boldsymbol{\beta}, \sigma|\mathbf{X}, \mathbf{hcb}) \propto \sigma^{-2N-1}\exp\{-[v^*s^{*2} + (\mathbf{Z}\text{-}\mathbf{Z}^*)'(\mathbf{G}'\mathbf{G})(\mathbf{Z}\text{-}\mathbf{Z}^*)]/2\sigma^2\} \qquad (18)$$

where

$$v^*s^{*2} \equiv (\mathbf{w}\text{-}\mathbf{GZ}^*)'(\mathbf{w}\text{-}\mathbf{GZ}^*)$$

(19)

If v* is set equal to N, the joint posterior p.d.f. in (18) is of the normal-gamma form

$$g(\mathbf{b}^*, \boldsymbol{\beta}, \sigma|\mathbf{X}, \mathbf{hcb}) \propto \sigma^{-N}\exp\{-[(\mathbf{Z}\text{-}\mathbf{Z}^*)'(\mathbf{G}'\mathbf{G})(\mathbf{Z}\text{-}\mathbf{Z}^*)]/2\sigma^2\}\cdot\sigma^{-(v^*+1)}\exp\{-[v^*s^{*2}]/2\sigma^2\} \qquad (20)$$

To get the ‘optimal’ estimator, the marginal posterior p.d.f. of **Z** is obtained by ‘integrating out’ σ from (20). The required integral is

$$g(\mathbf{b}^*, \boldsymbol{\beta}|\mathbf{X}, \mathbf{hcb}) \propto \int_0^\infty g(\mathbf{b}^*, \boldsymbol{\beta}, \sigma\,|\,\mathbf{X}, \mathbf{hcb})\, d\sigma \propto \left[1 + \frac{1}{v^*}(\mathbf{Z} - \mathbf{Z}^*)'\frac{(\mathbf{G}'\mathbf{G})}{s^{*2}}(\mathbf{Z} - \mathbf{Z}^*)\right]^{-N}$$

(21)

This marginal posterior p.d.f. for **Z** is a multivariate t-distribution, with mean **Z***, covariance matrix $[v^*/(v^*-2)]s^{*2}(\mathbf{G}'\mathbf{G})^{-1}$, and degrees of freedom parameter v*=N [10, p. 312]. Thus, assuming a quadratic loss function, the optimal estimator of **Z** is **Z*** as given in equation (17). Alternatively, since the t-distribution is unimodal and symmetric, **Z*** can be interpreted as the generalised maximum likelihood estimator. The parameter s^{*2} is obtained by dividing v^*s^{*2} in (19) by v*=N.

Using (13) and (14) in (17), the estimator **Z*** can be written in terms of $\mathbf{A}_1$ and $\mathbf{A}_2$ as

$$\mathbf{Z}^* = (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{w}$$
$$= \begin{pmatrix} (\mathbf{A}_1 + \mathbf{A}_2)^{-1}\mathbf{A}_1\mathbf{hcb} + (\mathbf{A}_1 + \mathbf{A}_2)^{-1}\mathbf{A}_2\mathbf{X}\{\mathbf{MX}'\mathbf{A}_2(\mathbf{A}_1 + \mathbf{A}_2)^{-1}\mathbf{A}_1\mathbf{hcb}\} \\ \mathbf{MX}'\mathbf{A}_2(\mathbf{A}_1 + \mathbf{A}_2)^{-1}\mathbf{A}_1\mathbf{hcb} \end{pmatrix} \qquad (22)$$

Thus, the Bayesian estimator of the optimal budgets **b*** (the first row of (22)) is a matrix-weighted average of the practices’ historical costs **hcb**, and the predicted value of **b*** from model (6), using $\mathbf{MX}'\mathbf{A}_2(\mathbf{A}_1+\mathbf{A}_2)^{-1}\mathbf{A}_1\mathbf{hcb}$ as the estimator of **β**. It is instructive to consider the form **Z*** takes when $\mathbf{A}_1$ and $\mathbf{A}_2$ are both assumed to be the identity matrix $\mathbf{I}_{N\times N}$. In this case, (22) collapses to

$$\mathbf{Z}^* = (\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{w} = \begin{pmatrix} \frac{1}{2}\mathbf{hcb} + \frac{1}{2}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{hcb} \\ (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{hcb} \end{pmatrix}$$

(23)

The estimator of **β** is just the Ordinary Least Squares estimator in a regression of **hcb** on **X**, and the estimator of **b*** is a weighted average as before, but with both weights equal to ½. More generally, if $\mathbf{A}_1 = k_1\mathbf{I}_{N\times N}$ and $\mathbf{A}_2 = k_2\mathbf{I}_{N\times N}$, where k_1 and k_2 are positive scalars, (22) takes the form

$$\mathbf{Z}^* = (\mathbf{G'G})^{-1}\mathbf{G'w} = \begin{pmatrix} \left[\frac{k_1}{k_1+k_2}\right]\mathbf{hcb} + \left[\frac{k_2}{k_1+k_2}\right]\mathbf{X}(\mathbf{X'X})^{-1}\mathbf{X'hcb} \\ (\mathbf{X'X})^{-1}\mathbf{X'hcb} \end{pmatrix}$$

(24)

Thus, when determining **b***, more weight can be given to historical cost information relative to ‘new’ information by setting $k_1 > k_2$, and vice versa. Note that the estimator of $\boldsymbol{\beta}$ is unaffected by changes in k_1 and k_2.

4. The data

To examine the empirical performance of the BRAM, the model was applied to 1993-94 data for 96 general practices in one English HA, *Lincolnshire Health*. This data set was chosen because it has already been thoroughly studied in the published literature [12], and therefore provides a familiar context within which to explore a new technique. The aim of the application was to use simple specifications of models (5) and (6) to produce an estimated ‘optimal’ 1994-95 budget for each practice in the data set, and to compare the estimated budgets with the actual allocations for that year. It should be emphasised that this was only intended as an exploratory exercise, using the minimum data required to obtain realistic results.

Lincolnshire is a rural county of East England, with approximately 550000 inhabitants. During the financial year 1993-94, the county had 108 general practices within its boundaries, although complete data were available for only 96 in the present study. Lincolnshire’s prescribing allocation for 1993-94 was 43.453 million pounds. However, there was an overspend in 1993-94 of almost 6.5 per cent, and a population rise of almost 1 per cent, which meant that actual growth in spending in 1994-95 had to be less than half of the allocated percentage uplift for that year (approximately 12 per cent) to avoid another overspend. This was viewed as a challenging target. It was particularly important in this climate to avoid wasting resources, and to set equitable practice-level budgets.

To implement the BRAM, data are required for **hcb** in (5), and for the **X** matrix in (6). In addition, assumptions must be made about the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ pertaining to the disturbance covariances in these models. The vector **hcb** was defined as the natural logarithm of Net Ingredient Cost (NIC) plus an 8 per cent uplift for each Lincolnshire practice in 1993-94. The NIC is the expenditure on prescribed drugs, excluding dispensing fees and any discounts offered. The 8 per cent uplift is in line with NHSE recommendations for this period. The mean and standard deviation of the 1993-94 NIC for the 96 practices in the sample are 420732.66 pounds and 237623.02 pounds respectively. The **X** matrix contained only two components for each practice. The first was a constant term, analogous to the intercept coefficient in linear regression analysis. The second was a variable measuring the total number of ASTRO-PUs pertaining to each practice (mean = 22973.11, standard deviation = 13480.89). The coefficient of the ASTRO-PU variable was expected to be positive, indicating that practices with a higher number of ASTRO-PUs should be allocated a larger budget.

The matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ were defined as (N×N) identity matrices multiplied by $k_1 = 1$ and $k_2 = 2$ respectively. Thus, the ASTRO-PU information was given twice the weight of the historical cost data in this exercise. The BRAM was implemented using a simple computer program in the econometric software package GAUSS for Windows NT/95, Version 3.2.37.

5. Results

Tables 2 and 3 show the estimated optimal budgets for the 96 practices (the antilogarithms of **b***), the estimated $\boldsymbol{\beta}$ coefficients from model (6), and the 't-ratio' of each estimate (defined as the ratio of the estimate to its standard error based on the joint posterior t-distribution of the parameters). Also shown in Table 2 are the actual 1994-95 budget allocations for the 96 practices in the sample.

All the estimated parameters have high 't-ratios' by conventional standards in classical statistics, in which an absolute cutoff point of approximately 2 is used to judge statistical significance. The t-ratios in Tables 2 and 3 are all considerably larger than 2, indicating that the estimates are relatively precise, but it should be emphasised that it is incorrect in a Bayesian context to interpret them as test statistics for the null hypothesis that the parameters are equal to zero. As expected, the coefficient of the ASTRO-PU variable is positive, indicating that practices with a higher number of ASTRO-PUs should receive a larger budget allocation.

Comparison of the actual and the BRAM budget allocations for 1994-95 in Table 2 suggests a high degree of correlation between the two (correlation coefficient = 0.9551). A 95 per cent confidence interval for the average difference between them is -17078.48 ± 18237.10. Since this interval includes zero, it is not possible to reject the null hypothesis that the two are equal (except for random error) at the 5 per cent level. The similarity of the actual and predicted budgets in Table 2 suggests that the budget setting procedure in Lincolnshire, involving the use of capitation formulas and bilateral negotiations, can realistically be replicated by using the BRAM. The historical cost and ASTRO-PU information appear to have been implicitly weighted in the ratio 1:2 in equation (24).

6. Conclusions

This paper has developed a Bayesian Resource Allocation Model (BRAM) for setting prescribing budgets in general practice. It offers two key advantages over existing formulas based on capitation, and other approaches involving bilateral negotiations. First, it allows all relevant data to be incorporated in a rational way in the budget setting process, including historical costs and ASTRO-PUs. It could also easily incorporate other 'needs' variables, and even information not normally used in budget setting, such as epidemiological data. In contrast, capitation formulas based on ASTRO-PUs are unduly wasteful of data, since they cannot use the potentially valuable information contained in routinely-collected historical costs, and might also miss other legitimate variables explaining differences in prescribing costs across practices. Approaches involving bilateral negotiations to try to control for relevant variables run the risk of misallocating funds due to strategic behaviour among practices, and to inconsistent use of available information from one case to the next.

Second, the BRAM forces budget setters to be explicit about the weight given to each type of information in the budget allocation process, since all uncertainty is formulated explicitly in terms of probability distributions. In contrast, existing approaches are not transparent, since they involve *ad hoc* adjustments to initial capitation-based allocations that cannot easily be scrutinised.

A simple application of the BRAM to 1993-94 data on 96 Lincolnshire practices can almost replicate the 96 actual budget allocations for 1994-95, suggesting that the cumbersome and costly process of ‘capitation plus negotiation’ can realistically be superseded by the Bayesian approach. However, prospective evaluations of the BRAM are required in the context of existing regulatory structures, using current data to estimate *future* optimal budgets.

Note that the BRAM offers a new approach to combining different types of information in the budget allocation process, but it does not solve the problem of what variables *should* be included. Recent research has shed important new light on this issue [7], and more research along these lines may be required to maximise the usefulness of the multivariate Bayesian approach developed here.

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Table 1. ASTRO-PU weighting system

Age	Male PU value	Female PU value
0-4	1	1
5-14	1	1
15-24	1	2
25-34	1	2
35-44	2	3
45-54	3	4
55-64	6	6
65-74	10	10
75+	10	12
Temporary Residents	0.5	0.5

Table 2. Estimated optimal 1994-95 budgets from the Lincolnshire BRAM

Practice	Estimated 1994-95 budget	't-ratio' (ratio of estimate to its standard error)	Actual 1994-95 budget
1	761033.32	94.78	787210
2	225450.44	87.34	201077
3	245019.15	88.16	239651
4	249628.75	88.31	264060
5	633737.22	93.82	506222
6	258730.50	88.57	259427
7	203734.40	86.52	183602
8	437174.29	92.74	514146
9	377987.79	91.72	362408
10	288955.18	89.49	329454
11	654478.60	94.47	649134
12	283013.11	89.35	288677
13	242655.71	88.08	244753
14	302692.51	89.91	363090
15	196936.11	85.89	212681
16	240173.96	87.88	245613
17	569921.42	94.25	647826
18	294692.13	89.76	327888
19	236776.54	87.89	246545
20	407577.87	92.26	461234
21	1123593.59	95.32	995812
22	281152.76	89.34	280813
23	641019.82	94.34	598157
24	584710.72	94.12	581213
25	226119.14	87.40	215503
26	701706.63	94.61	660640
27	249276.24	88.27	232306
28	207568.89	86.97	159422
29	233484.10	87.62	241374
30	215059.92	86.96	220317
31	504821.30	93.49	497279
32	262974.98	88.78	279824
33	377968.11	91.72	388147
34	309988.85	90.22	374180
35	224167.76	87.62	260741
36	200930.14	86.39	162569

Table 2 (continued).

Practice	Estimated 1994-95 budget	't-ratio' (ratio of estimate to its standard error)	Actual 1994-95 budget
37	1528816.22	93.27	988254
38	314525.61	90.33	319078
39	488869.74	93.20	438356
40	233870.91	87.67	232796
41	142309.56	83.25	92977
42	220720.54	87.20	208555
43	425890.27	92.57	489002
44	97419.30	80.15	50802
45	396270.42	92.03	378648
46	352047.45	91.21	350911
47	205212.44	86.62	183523
48	606360.36	94.15	620115
49	321860.34	90.46	357643
50	293406.92	89.76	336187
51	203369.77	86.51	205079
52	486327.04	93.31	445026
53	320113.51	90.45	358180
54	191447.40	85.87	165753
55	999949.54	95.07	978406
56	430538.13	92.64	490571
57	666125.58	94.73	737972
58	178979.28	85.12	153577
59	540882.86	93.35	512710
60	278418.28	89.09	265546
61	494104.71	93.32	512890
62	318723.03	90.29	383921
63	306248.71	90.15	289023
64	205319.59	86.71	168854
65	471048.91	93.12	538651
66	215726.19	87.02	223623
67	116187.40	81.61	55930
68	510575.14	93.30	435024
69	631618.29	94.63	707909
70	330093.47	90.73	326872
71	195040.88	85.92	163467
72	148155.86	83.78	106121

Table 2 (continued).

Practice	Estimated 1994-95 budget	't-ratio' (ratio of estimate to its standard error)	Actual 1994-95 budget
73	682042.28	94.42	642792
74	498889.23	93.53	577906
75	587434.35	94.03	532218
76	466998.73	93.12	524387
77	414971.02	92.39	492033
78	262978.44	88.91	253459
79	489125.13	93.13	427357
80	359947.97	91.35	425030
81	143380.81	83.36	147942
82	464738.71	92.87	445677
83	287077.88	89.55	253006
84	797595.02	94.74	658789
85	217640.78	87.11	199235
86	381366.26	91.78	364965
87	666611.41	94.27	590280
88	1478568.93	93.97	1067395
89	310803.03	90.32	282780
90	771190.94	94.81	726207
91	983896.89	94.71	878454
92	1383525.90	93.86	1009076
93	458647.94	93.00	488993
94	327938.42	90.66	358833
95	252961.98	88.47	230064
96	265404.96	88.87	261365

Table 3. Estimated β coefficients from the Lincolnshire BRAM

Variable	Coefficient	't-ratio' (ratio of estimate to its standard error)
Constant	0.06	199.15
ASTRO-PUs (10000s)	0.02	18.10