Draft version

Improving geographic equity: A location-allocation model to redistribute hospital supply

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Abstract: *A common problem for countries with national health services is that there are inequities in utilisation, which reflect inequities in supply. The pursuit of geographic equity objectives requires information on how to change the distribution of supply to achieve greater equity in utilisation. Previous methods for analysing hospital changes have relied on crude assumptions on patients' behaviour and neglected the interaction between hospital size and the levels of utilisation of alternative hospitals. This paper reports a new approach to the problem. This approach combines mathematical programming and econometric techniques in a location-allocation model. A two-part econometric model predicts utilisation by small areas and hospital sites using a large set of supply, demand and spatial variables. The econometric model is used as a constraint in the mathematical program, as a predictor for population behaviour for changes in hospital supply. The objective function of the mathematical programming model uses an equity index that summarizes deviations between the logarithm of utilisation; and a set of lower and upper bounds on the levels of hospital redistribution are imposed as constraints of the mathematical program. This model has been applied on Portuguese hospitals. Preliminary results from the combined model show: higher redistribution of hospital supply towards the south and interior of the country; hospital supply is a limited tool for influencing spatial patterns of hospital utilisation; and gains in location accessibility are affected by losses in utilisation. Further empirical results are: primary care is acting as substitutive from public hospital care; and each central hospital has a specific role inside regional health systems.*

Keywords: geographic equity, hospital sector, location-allocation, redistribution

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Introduction

A common problem for countries with national health services is that there are inequities in utilisation, which reflect inequities in supply. The pursuit of objectives of geographic equity requires information on how to change the distribution of supply to achieve greater equity in utilisation. Also, many countries have reached acceptable levels of hospital supply and are now concerned with introducing improvements to the current network (Edwards, Hensher, and Werneke 2000). This implies that redistribution is required in a 'zero sum context', as most countries are not levelling up hospital resources. This study is concerned with methods to inform how to improve geographic equity of utilisation, using as policy tool the geographic redistribution of hospital supply.

The characteristics of the health care system influence the choice of methods for analysis. This is a study of a NHS system in Portugal where: a hospital network is dominated by public provision[3]; there is central planning of hospital supply; geographic flows are influenced by a wide set of variables, such as doctors' choices, institutional structures (such as the gate-keeping system) and patients' preferences; and where public hospital services are nearly free at the point of service (e.g., ability to pay is not seen as a key factor in explaining hospital use). The Portuguese system has much in common with that of the U.K. and Southern European countries (such as Spain and Italy) (Mossialos and Le Grand 1999).

The problem of deciding upon locations and capacities between facilities located in different geographic points is treated in geographic and operational research literature as one of location and location-allocation modelling. Location-allocation models endeavour optimally to locate systems of facilities and allocate simultaneous demand to them (Hodgson, Rosing, and Storrier 1996). Location models optimally locate systems of facilities for a defined set of providers objectives, and consumers' responses to location factors are made independently of provider conditions (Rushton 1987). Location and location-allocation models differ on what they interpret consumers' decisions over the choice of provider (Love, Morris, and Wesolowsky 1988). As the behaviour of patients towards changes in hospital location and dimension is a key element for analysing variations in hospital supply, the focus should be placed on location-allocation models.

A location-allocation model requires four main decisions on the approach to modelling (Ghosh and Rushton 1987): decision on the objectives to pursue; assumptions of consumer spatial choice; representation of the environment[4]; and choice between deterministic or stochastic models. Most of these analytical choices entail judgements (Mandell 1991). Three main methods represent variations in these four elements and have been used for analysing geographic distribution and redistribution of public facilities (including hospital supply): spatial interaction, entropy and mathematical programming models. This paper describes how these methods operate.

Spatial interaction models (such as gravity models) constitute a form of probability interaction modelling (O'Kelly 1987), and consider three main variables in spatial analysis: population numbers, hospital size and distance. They make use of decay parameters calibrated from empirical data, and as strength, they have been used as reliable models for replicating the current pattern of patient flow between demand and treatment zones (Cho 1998). The main weakness of

[3] Note that the objectives to be pursued in location-allocation models differ for the public and private sectors (Current, Min, and Schilling 1990) (Erlenkotter 1983).

[4] This is the case for the choice of the geographic level of analysis, and of travelling times between geographic points.

gravity models is their lack of reliability for predicting user flows (Porell and Adams 1995). Despite this, they have been used for that purpose (Hallefjord and Jornsten 1984) (Mayhew, Gibberd, and Hall 1986) (Brown 2001). Their predictive problems arise because they assume that when there are changes in one hospital, all the other hospitals gain in proportion to their shares of utilisation prior to that change, but empirical evidence shows a violation of that assumption and "an inherent instability in the spatial choice rule represented in the model" (McLafferty 1988). Gravity models present other weaknesses. As gravity models operate at the aggregate level, they do not consider specificities from the local level, and do not consider the inherent spatial organization of hospitals (for example, they do not consider whether there are hospitals of reference for a population area). Further, the estimation of decay parameters demands a choice of functional, together with the estimation of the decay function for different groups of hospitals, which entail methodological problems (McLafferty 1988).

Entropy models (EMs) have been used in different contexts and disciplines (such as information theory (Arndt 2001))[5] and are a type of mathematical programming model that makes use of the first principle of data reduction (Wu 1997): when there are incomplete data, the solution must include and be consistent with all relevant available data. EMs' strengths are: their adequacy when there is a lack of insights on what to include in the model (Anas 1983); and they avoid the economic determinism of micro-economic models, may be rationalized theoretically, and examine only small components of the decisions of individuals (Webber 1978). As weaknesses, entropy models follow a holistic view that imposes institutional and system constraints, and do not consider the way groups make spatial choices (Nijkamp 1978).

Mathematical programming (MP) models have been widely used for locating and allocating public facilities. They maximise some kind of equity concept (for example, distance travelled by patients), assume some type of user behaviour (for example, patients travelling to the closest hospital), and incorporate some specificities from health care systems. Table 1 presents some choices of MP models. Their strength relates with their flexibility for the choice of an objective function, for providing an optimal solution and for modelling constraints in the system; and their weaknesses are related with the use of crude assumptions on users behaviour.

In the health care sector context, these three methods have been inappropriate in two areas. First, most of the models in use are location models, which have assumed simple rules of allocation of demand and have not captured adequately patients' (or doctors') behaviour. The use of crude assumptions on the rule of patients' choice of hospital means that they cannot be used to predict changes in utilisation (Bennet 1981) (Mohan 1983) (Rushton 1987) (McLafferty 1988) (Avella et al. 1998). Second, gravity models do model interaction but in an unsatisfactory way; and the other groups of models ignore interactions between hospital size and the levels of utilisation of alternative hospitals (Porell and Adams 1995)[6]. This is critical in the context of prediction of flows. This papers reports a new approach to the problem.

[5] EMs have been used in areas such as thermodynamics, statistical dynamics, statistics and information theory (Wu 1997) (Fang, Kajasekera, and Tsao 1997) (Arndt 2001).

[6] Previous methods have made the assumption of independence of irrelevant alternatives i.e., flows to one hospital do not depend on the set of alternative hospitals (Porell and Adams 1995).

Table 1: variety of mathematical programming models for public and health care facilities location

MP models under several classifications	Description	Examples from literature
Single- vs. multiple objective models	Single objective models use a single objective: minimise the distance from users to the closest provider point or minimise the maximum distance; or models that minimise the total travelling costs. Multiple-objective MP models pursue several objectives at once; some examples of models are models based on goal programming techniques, multi-criteria utility models and bi-criteria models[7].	Single-objective (Mohan 1983) Multi-objective (Rushton 1987)
Optimising vs heuristic models	Optimising models provide an optimal solution. Heuristic models produce satisfactory results that might not be an optimal solution.	Optimising models (Mohan 1983) Heuristic models (Bennet 1981) (Walsh, Page, and Gesler 1997)
Nested vs. hierarchical models	Hierarchical models represent a referral system, such as top-down and bottom-up models of referral formulation; or account for total travel costs incurred in accessing different levels of health care facilities. Nested models treat all the facilities of the hierarchy at the same level, so that facilities from higher levels provide the services from the lower levels.	Hierarchical (Rahman and K. 2000) (Church and Eaton 1987) Nested models (references in (Marianov and Serra 2001)).

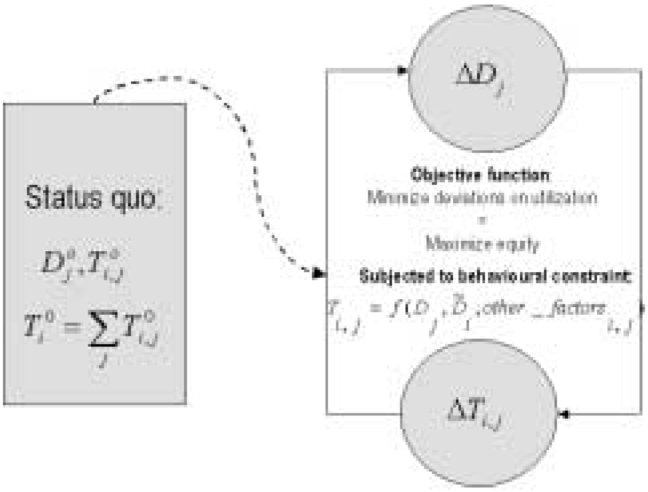
The approach developed here combines mathematical programming and econometric techniques in a "flow-demand" location-allocation model. "Flow demand" is defined as a stream between origin and destinations, inside a system that interacts as a whole, and where the flow terminates in the destination site (Hodgson, Rosing, and Storrier 1996). Applying the "flow demand" concept to this study, utilisation is treated as a flow between population points and hospital sites; and utilisation flows are taken as the dependent variable (whereas the convention is to use utilisation per area); and flows are the feature that allows for modelling interaction. A two-part econometric model is used to predict flows (by small areas and hospital sites). This uses a large set of supply, demand and spatial variables; and integrates a geographically-based alternative hospital index. The two-part model takes account of the mixed distribution of flows data. Both parts of the econometric model are used as constraints in the mathematical program, as predictors of population behaviour with respect to changes in hospital supply. The MP model minimises a specific equity index that summarizes deviations between the logarithm of utilisation flows. This approach is applied to the Portuguese health care system. The rest of this paper has five sections which: present the components of the location-allocation model; explain the two-part econometric model; present the proposed mathematical program; apply the complete model to the Portuguese hospital system; and discuss the strengths and weaknesses of the approach.

A location-allocation model to redistribute supply

A mixed modelling strategy was chosen in order to combine the advantages of a behavioural econometric model to simulate patients' behaviour and a MP model to determine optimal capacities and flows. The model is applied to the macro level of a country, and must use of data at the small area level. The MP model relates the elements shown in figure 1.

[7] Goal programming models allow decision-makers to assign weights to the realization of each goal that is included in the objective function and perform sensitivity analysis. Multi-criteria utility models quantify trade-offs and test alternatives under changes in the objective function (Cho 1998) (Mayhew and Leonardi 1982)). Bi-criteria models use a constraint approach to multi-objective programming (Mandell 1991).

Figure 1: simultaneous spatial distribution of hospitals and flows of utilisation



Where:

$T_{i,j}$ is the flow of patients between population i and hospital site j

D_j is the size of hospitals on site j

$\tilde{D}_i$ is the alternative hospital supply index for population in *i*

$other\_factors_{i,j}$ is the other set of determinants that influence utilisation flows[8]

The MP model optimises an equity index by minimising deviations on utilisation against a target and by using hospital supply as the decision variable (the D_j's). The model's structure is designed: to accommodate information on the behaviour of spatial demand (captured by equation 1); and to integrate a set of lower and upper bounds on the levels of hospital redistribution. These bounds are set so that marginal changes in the system are to be analysed (this is the context of countries with a sufficiently sized network of hospitals but with geographic inequalities).

$$T_{i,j} = f(D_j, \tilde{D}_i, other\_factors_{i,j}) \qquad (1)$$

Econometric modelling is chosen to capture patients' behaviour (to be used for prediction). This option follows previous studies that have used binary choice models for explaining joint origin-destination flows (Anas 1983). As described below, the choice for a two-part model was based on the mixed nature of distribution of data on utilisation flows. A stochastic component is used to capture small area variations. The econometric model was constructed to be linear within a mathematical programming model, such as to avoid difficulties by the introduction of non-linear relationships between variables. The hospitals were modelled inside a nested system (Marianov and Serra 2001) as defined in table 1, to generate estimates of size for hospitals that occupy different places in the hierarchy. The two-part econometric model is built as a behavioural model that can be derived from hypothesis of utility maximisation on patients' use of hospital services (assuming the same pattern of utilities across areas). It includes hospital sector variables and other variables from the health care system that influence hospital utilisation, and tests for the role of income, socio-economic, private care, primary care and social care indicators on the hospital sector. Previous studies have failed to account for most of these factors. This model can be interpreted from the viewpoint that either the patient or the physician can influence utilisation.

In order to solve the problem of interaction between hospital size and the levels of utilisation of alternative hospitals, two key elements in the design of the models were: the use of flows as the dependent variable (as described above); and use of an index of alternative hospital supply to a population area. The hospital alternative supply index is an independent variable that is to be used to explain utilisation flow for a specific hospital. Thus, the econometric model aims to

[8] These factors can still be further segmented in three groups: determinants that vary between population areas; determinants that vary across population sites; and determinants that vary between population areas and hospital sites.

isolate the impact of hospital supply and alternative supply on utilisation flows, after controlling for variations on other factors, as shown in equation (1).

Results from the integrated model are analysed under the framework presented in table 2. Utilisation, distance and supply levels are indicators of equity of access. Redistribution levels are interpreted as a measure of how manageable is implementation of changes. Variations in district levels of supply are compared with needs-based estimates of supply, as produced in another study (Oliveira 2000).

Table 2: framework for analysis of outputs

Indicator type		Inequality measure
Utilisation	i.	Utilisation rates by small area
	ii.	Total utilisation level in the system
Distance	i.	Average distance by small area
	ii.	Average distance in the system
Supply	i.	Proposed variation in the levels of supply at the district and health region level
	ii.	Ranking of hospitals by potential of improvement on (equity of) utilisation
Redistribution	i.	Levels of redistribution (geographic redistribution of winners and losers)

The behavioural model

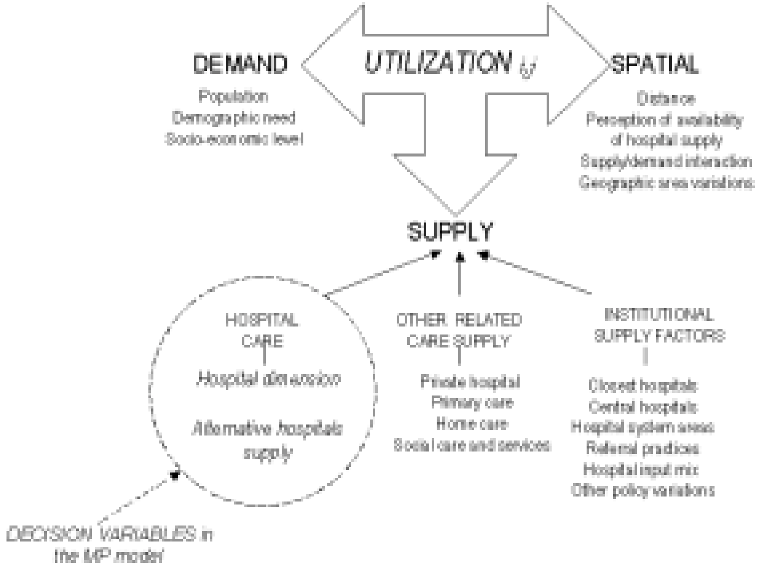
The reasoning behind the choice of a behavioural model was explained above. This section gives details of the econometric model: the factors that influence hospital flows; and the implications of having flows as the dependent variable.

Conceptual elements

The factors that influence hospital flows are categorized in three classes: demand, spatial and supply (see figure 2). Supply is further decomposed in three main components: hospital supply (from a hospital and from alternative public hospitals), alternative and competing supply to public hospital care, and supply institutional factors. This follows previous location-allocation models that have focused on four variables (population numbers, hospital size, distance, and on hospital roles in the hierarchy) (Porell and Adams 1995). Previous studies have shown that utilisation is affected by hospital size, physician affiliations and service mix on the patterns of hospital use (McLafferty 1988)[9]. An application of this type of model provides estimates on the relationship between public hospital care and other health and social services.

[9] The expected relationships between covariates and utilization flows are hypothesised below in table 3.

Figure 2: determinants for hospital utilisation flows between a population area and a hospital site



The econometric model

The specific characteristics of flow utilisation data have constrained the choice of econometric model. As other utilisation indicators (such as number of medical visits, medical expenses, etc), the distribution of utilisation flows data has a mixed nature and the following characteristics (Blough, Madden, and Hornbrook 1999): flows are nonnegative, there is a high proportion of zero flows, positively skewed empirical distribution of the non-zero flows (corresponding to a heavy-tailed distribution), and non-constant variance[10],[11]. Further, the model to be used has to produce outputs that can be analysed in their natural unit.

A two-part model is used for explaining flows because these can be seen as a two-stage process of decision-making (Pohlmeier and Ulrich 1994) (Mullahy 1998), and is appropriate when there are upper tails and/or high-end outliers (Mullahy 1998). The two-part model is represented in equation 2 (being $y = T_{i,j}$ an utilisation flow, and x a set of covariates).

$$\mathrm{E}[y/x] = \Pr(y > 0/x) * \mathrm{E}[y/y > 0, x] = \Pr(T_{i,j} > 0/x) * \mathrm{E}[T_{i,j}/T_{i,j} > 0, x] \qquad (2)$$

With *Part* $A = \Pr(T_{i,j} > 0/x)$ and *Part* $B = \mathrm{E}[T_{i,j}/T_{i,j} > 0, x]$. *Part A* represents a dichotomous model to predict the probability of population from area i making use of hospital site j. *Part A* might also be interpreted as a model for populations' choice for the set of hospitals for utilisation. Logit and probit models were used to estimate *Part A* of the model (Mullahy 1998).

Part B represents a model to predict the level of flow, given that the expected level of flow is greater than zero. The choice of econometric technique for this part of the model must specifically consider the count nature of utilisation flows data and the heavy tail of its distribution (which requires a procedure to stabilise the variance). The appropriate methods for data with

[10] For the Portuguese case, utilization flows (information taken from the database built with the characteristics described below) presents as statistics: 18700 observations; around 85% of zero observations; sample mean of 47; standard deviation of 48; maximum value of 70674; skewness of 57; and kurtosis of 4881.

[11] Previous location studies have often ignored the impact of zero flows on the estimation techniques (Porell and Adams 1995).

these characteristics are (Blough, Madden, and Hornbrook 1999)[12]: log-linear model estimated by ordinary least squares (OLS); or estimation under a generalised linear model (GLM), with a log link. The log-linear model implies severe problems in estimation when the initial model is heteroscedastic (Manning 1998)[13]; and whether or not there is heteroscedasticity, it is necessary to follow a retransformation procedure from the log to the natural scale of utilisation. Retransformation is mathematically difficult (Blough, Madden, and Hornbrook 1999) and if not correctly applied, produces biased estimates (Manning and Mullahy 2001). The GLM model was chosen as it is a flexible technique that allows for analysing data in their natural scale without any need for retransformation, is more appropriate when there is evidence of heteroscedasticity, and makes use of minimal assumptions[14]. The Poisson family distribution was assumed to describe the behaviour of the error distribution of the GLM model, on an *a priori* basis, in accordance with the count and discrete nature of flows data (McCullagh and Nelder 1983). An extended Park test proposed by Manning and Mullahy (Manning and Mullahy 2001) was carried on the raw-scale residuals to check for the adequacy of the use of the Poisson distribution.

The determinants for the two parts of the econometric model might differ[15]. For capturing the differentiated role of central hospital sites, alternative specifications were tested using both dummies and decomposable variables for hospital size, in an attempt to capture their specificities[16].

The mathematical programming model

The structure of the MP model comprises five features:

- The model maximises an equity function that aggregates geographic data.
- It is a multi-hospital model, with several facilities to be located and their size estimated simultaneously.
- It is assumed that there is a pre-determined, discrete and finite set of potential locations, corresponding to the status quo (current sites of hospital supply).
- Total supply is constrained to the current level, and upper and lower bounds of variation of supply are imposed so as to maintain the (marginal) redistribution character of the model.
- Utilisation flows are generated by the two-part behavioural model, but are not subjected to any constraint at the individual level.

The next section explains the reasoning behind the choice of the objective function and the model structure.

Objective function

An objective function contains an equity index. There are various possible indexes for summarising geographic deviations (Erkut 1993) (Marsh and Schilling 1994) (Kostreva and Ogryczac 1999) (Ogryczak 2000). The location literature has been using crude objectives, such as minimisation of patients' accessibility costs or maximisation of demand orientation (Current,

[12] As discussed later, one must be aware that log model results are about geographic means (Manning 1998), and on the consequences of this in interpreting the results.

[13] In the application to Portuguese hospitals, the running of a preliminary log-linear model for the second part of the model has shown strong indications of the presence of heteroscedasticity. Though, in the presence of heterostecedasticity, the log-linear model can still be applied (Manning 1998) (Duan et al. 1983).

[14] Nevertheless, GLM estimation might be imprecise (but consistent) if the log-scaled errors are heavy tailed (Manning and Mullahy 2001). Control has to be made for this.

[15] The two parts of a two-part model might have distinct (economic, political, geographic) determinants, as well as different political relevance (Mullahy 1998).

[16] In addition, choice between alternative GLM-based models was based on the criteria: predictive power, parsimony, intuitive explanation of coefficients, specification tests, and properties of residuals and deviance.

Min, and Schilling 1990). Different indexes have implicit different preference models and their choice depends on the context of analysis. Given the nature of utilisation flow data and the information provided by the two-part model, none of the available indexes was suitable for this study –all the indexes available aggregated one-dimensional information, such as utilisation per population area, and no bi-dimensional index is known (i.e., by population area and by hospital site). Also the second part of the econometric model produces data on the logarithm unit, which must be considered in the formulation of the MP model.

An equity index should summarize deviations between utilisation flows with some (more equitable) target distribution inside an index structure and ought to satisfy properties often cited in the literature on location of facilities (Marsh and Schilling 1994). These include: analytical tractability for problem size and computational requirements; appropriateness for interpretation; not discriminating between the (geographic) groups being evaluated; and the principle of transfers, in which the equity measure decreases as transfers from the best to worst-off groups occur. Following the variance of the logarithms index version presented in Marsh and Schilling (Marsh and Schilling 1994), formula (2) is created:

$$\frac{\sum_i \sum_j \left(\log U_{i,j} - \log U_{i,j}^r\right)^2}{N} \tag{3}$$

with: $\log U_{i,j}$ as the logarithm of utilisation flow between population in i and hospital site j, $\log U_{i,j}^r$ is the utilisation distribution target towards which one attempts to progress[17], and $N = n * m$, being $i = 1,...,n$, and $j = 1,...,m$.

The objective function is to minimize:

$$\sum_i \sum_j a_{i,j}^2 \text{ with } a_{i,j} = \log U_{i,j} - \log U_{i,j}^r = \log\left(\frac{U_{i,j}}{U_{i,j}^r}\right) = \log\left(\frac{\hat{p}_{i,j} * U_{i,j}^{'}}{U_{i,j}^r}\right) = \log \hat{p}_{i,j} + \log U_{i,j}^{'} - \log U_{i,j}^r \tag{4}$$

where: $\log \hat{p}_{i,j}$ is the logarithm of the probability of use, taken as fixed in this study –this assumption is accepted in the context of a model that is to be used for (marginal) redistribution; and $\log U_{i,j}^{'}$ is the level component of flows, as determined by the two-part of the econometric model.

Two alternative equity targets distributions were generated. These targets were based on underlying estimates of an equitable distribution of utilisation by populations, in accordance to:

a) A distribution that allocates population users to the closest hospital;
b) And a distribution that allocates users to three hospitals –the closest hospital, the second closest hospital, and the central closest hospital. This distribution is generated by a model that makes use of empirical findings that populations in small areas tend to make use of at most five hospitals, including a central hospital that is the unique provider of a set of services. In

[17] The proposed index is a measure of dispersion of the log of utilization flows (against an equity target). Theil (Theil 1967) (in the context of income) has shown that the variance decomposition is useful when the variable is approximately lognormal distributed. Note that the use of the variance of logarithms implies that the utilisation of an area is the geometric mean of the utilisation flows with origin on that area, while the use of the arithmetic mean would be more convenient and meaningful (Theil 1967). This should be taken into account in the analysis of results.

this model patients are allocated to the closest hospitals, they minimise distances, and the shares of patients to be allocated to each of the three hospitals are previously defined[18].

This objective function requires the use of quadratic programming. A quadratic function is a non-linear function, which implies concerns for assuring that the solution of the mathematical program is optimal[19]. The quadratic function needs to be convex so that the solution of the MP program generates not only a local optimum, but also a global optimum (Williams 1993a). As all the coefficients of the squared utilisation variable of expression (4) ($a^2_{i,j}$) have positive coefficients, the Kuhn-Tucker conditions that guarantee for global optimality are satisfied (Williams 1993b).

Model structure

The model uses as nonnegative decision variables: $\log U^{'}_{i,j}, D_j$ and $\tilde{D}_i$; and minimises the sum of the squared differences between the logarithm of utilisation flows and the logarithm of the target flows (shown in equation 4).

It uses four constraints. First, it uses a behavioural equation (taken from the second part of the two-part model). The formulation of this constraint depends on the specific econometric application obtained from empirical data[20]:

$$\log U^{'}_{i,j} = f(D_j, \tilde{D}_i, other_{i,j}), \forall i, j \qquad (5)$$

with $other_{i,j}$ being the parameter that captures the impact of other factors on hospital utilisation flows (treated as fixed in this model). Second, lower and upper bounds for changes on supply are imposed:

$$D_j \geq \min\_D_j, \forall j \qquad (6)$$

$$D_j \leq \max\_D_j, \forall j \qquad (7)$$

where $\min\_D_j$ is the minimal level of hospital size, computed under $\min\_D_j = fix\_capacity * cur\_D_j$, with $fix\_capacity$ as the percentage of capacity from current hospital sites to remain unchanged; cur_D_j is the current size of hospitals in site j; and where $\max\_D_j$ is the maximum level of hospital size, computed under $\max\_D_j = \max\_change * cur\_D_j$, with $\max\_change$ as the percentage of maximum size, as a function of current size.

Third, there is a constraint on the maximum level of capacity to be used in the system:

$$\sum_j D_j <= D \qquad (8)$$

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18 The specificities of this model and its programming structure might be requested to the author. The three closest hospitals model departs from the structure of the classical model that minimises the population weighted distance, when quotas for the use of the closest, second closest and to the closest central hospitals are previously defined. This model answers to empirical evidence that contradicts the nearest-center rule of patients' behaviour, mainly due to diversity in hospital supply (O'Kelly 1987).

19 Constraints of the quadratic mathematical program are linear (as seen in equations 5-8).

20 The right hand side of equation 5 is linear on the selected components).

where: $D = \sum_j cur\_D_j$ is the total capacity of the hospital system.

Application to the Portuguese hospital system

The Portuguese supply of hospital acute care measured by per capita indicators is about the OECD average (OECD 2000). Hospital supply is highly concentrated in coastal and urban areas (Oliveira 2000), resulting in high variation and hence inequities in utilisation of services by populations[21]. This distribution reflects past decisions on hospital supply, which have been made without regard to their inequitable consequences. The models described in this paper aim to give information on directions for change in the hospital system to achieve greater equity.

The organization of hospitals and gate-keeping system influence flows of patients to hospitals. Public hospitals are grouped in three categories: central and specialised hospitals, general district hospitals and district hospitals level I (DGS 1998). Central and specialised hospitals provide highly specialised services with advanced technology and specialist human resources, and are mostly located in three urban centres (Lisbon, Porto and Coimbra). District hospitals provide a range of specialist services, and are located at the district capital. At the base of the hierarchy, district level I hospitals provide internal medicine, surgery and one or two other basic specialties. This administrative division and the distribution of supply imply that the hospital system is divided in three hospital subsystems, corresponding to the north, centre and south of the country that have as centre supply points Lisbon, Porto and Coimbra, respectively.

Under the instituted gate-keeping system, patients can choose their GP (and specialist doctors after the GP consultation), or contract out doctors from a list (Mossialos and Le Grand 1999). Nevertheless, the gate-keeping system does not work perfectly (Pinto and Oliveira 2001): there are many exceptions to this referral scheme from civil servants, personnel of the military forces and bank employees; there is lack of co-ordination between GPs and specialised doctors; and a non-rationalised demand has motivated an abusive use of emergencies as an entry point to secondary care.

Consequently, flows of patients from population points to hospital sites are expected to reflect not only supply levels, but also the institutional structure of the hospital system, patients' preferences and accessibility costs. This section describes the database and presents results of econometric and mathematical modelling.

Database

The *concelho* level is used as the small area unit of geographic analysis: this corresponds to the smallest administrative unit for which statistics are available, and is a good basis for capturing changes in geographic accessibility. The dataset of 18700 observations links population geographic points (275 concelhos) with hospital geographic sites (68, as available in 1999); uses utilisation data taken from the hospital discharges DRG informational system (nearly 1 million discharges in 1999); and integrates data from a broad set of additional sources, on socio-economic indicators, primary care utilisation, private hospital supply, areas classification in regional health systems, Euclidean distances, etc[22]. The covariates included in the database and their expected sign in the econometric regressions are presented in table 3. A set of covariates makes use of dummies, in order to capture the institutional factors of the system.

[21] Portugal has a NHS-based health care system, and equity of access and geographic equity are explicit objectives in the political system.

[22] There were no data at the small area unit on welfare services and welfare financial flows, on hospital input mixes and on referral practices, which would improve the model.

Table 3: covariates included in the database (excluding interaction terms) and hypothesised behaviour

Conceptual explanatory variable	Indicators	Sign of first derivative	Sign of squared coefficient
Population	Resident population (i)	+	-
Demographic need	Differential costs implied by population age structure (computed using estimates of a age cost curve) (i)	+	
Socio-economic level	Purchasing power index Illiteracy rates	+/-[23] - /+[24]	
Distance	Euclidean distance (i,j)	-	+
Perception of availability	Accessibility coefficients from a gravity model (i,j)	+	-
Supply/demand interaction	Population*discharges (i,j)	+	
Geographic area variations	Dummies to the health district or health region level (i)	n.d.	
Hospital size	Discharges (j) Number of hospital units in the site(j)	+ +	-
Alternative hospitals supply	Alternative hospital supply index (i)[25]	-	
Private hospital supply	Private supply as measured by number of beds in the population site (i)	-	
Primary care supply	Primary care utilisation (i)	+ if complementary/- if substitutive	
Closest and central hospitals	Dummy if closest hospital (i,j) Dummy if closest hospital(i,j)*Discharges (j) Dummy if second closest hospital (i,j) Dummy if second closest hospital(i,j)*Discharges (j) Dummy if closest central hospital (i,j) Dummy if closest central hospital(i,j)*Discharges (j)	+ + + + n.d.[26] n.d.	
Hospital system areas	Dummy if Lisboa central hospital for population from the south region (i,j)*Discharges(j) Dummy if Porto central hospital for population from the north region (i,j)*Discharges(j) Dummy if Coimbra central hospital for population from the centre region (i,j)*Discharges(j)	n.d. n.d. n.d.	

Econometric regressions

The following methodological choices were made:

- Hospital discharges were taken as a proxy for hospital size. This is supported by evidence on: beds are a bad indicator of supply, as bed occupancy rates are low in comparison with rates from other NHS countries (OECD 2000), and beds' use has been constrained by the availability of human resources (Oliveira 2000);
- Use of Euclidean distances, as there were no data on travelling times[27];
- And test for the statistical significance of multiplicative terms across some variables (distances, population and hospital size)[28].

[23] This must be read as: + if higher income implies better accessibility to hospital care, and this offsets the impact of high income on a lower need for hospital care or on a higher use of private hospitals; - for the inverse.
[24] This must be read as: - if the effect of better education implies higher utilisation; and + if lower education implies higher use (*ceteris paribus*).
[25] Index formula: sum of other public hospital supply within 25 kms over sum of population within 25 km from the population site.
[26] It depends on the relationship between hospital size and the role of central hospitals.
[27] Estimates for travel costs or travel times are preferred to crow-fly distances, both from a theoretical and practical viewpoint (Taket and Mayhew 1981). Several studies have computed travel times under different methods, such as: road distances making allowance for different types of road (Bevan and Waring 1983) (Bevan and Waring 1986) and journey times (Weiman 1982). These studies have concluded that travel times are highly correlated with distance; and that the impact of using them on the gravity model, in comparison with crow-fly data, is negligible.

Problems in econometric modelling included multicollinearity (as expected (McLafferty 1988)) and difficulties in capturing the specific role of hospital sites with central hospitals. There is multicollinearity because many determinants of hospital utilisation influenced each other (in particular population, purchasing power, private care and socio-economic status; and between hospital discharges and number of specialties available[29]). Capturing in the model the specific characteristics of the three hospital sites with central facilities required the use of alternative models playing with different assumptions on fixed (i.e., dummies) and variable effects as covariates (in the GLM model). Results from the two-part econometric model are given in tables 4 and 5[30]:

Table 4: first part LOGIT model

Indicator	Variable	Coefficient	Z
Other	Constant	-.6918692*	-1.35
Distance	Distance(i,j)	-.0191592*	-6.56
Perceptions on availability	Gravity accessibility index(i)	1.431269**	0.75
Demand	Population (i)*Demographic need(i)	.0000116*	15.83
Primary care	Primary care utilization(i)/population(i)	-.5594*	-6.32
Supply availability	Discharges(j)	.0001331*	14.86
Alternative public hospital supply	Hospital competition "index" (i)	-.4962825*	-7.24
Institutional factors	Dummy for population in the north using the Porto hospital site(i,j)*discharges(j) Dummy for population in the centre using the Coimbra hospital site(i,j)*discharges(j) Dummy for population in the south using the Lisbon hospital site(i,j)*discharges(j)	-7.887485* -4.920875* -16.05501*	-9.00 -7.45 -8.92

Model summary: 18700 observations

Diagnosis: 94.97% correctly predicted classifications (0.5 cut-off); Pseudo R2= 63.67%; Wald Chi2(9): 2207.

*- Statistically significant at 1% level; ** Statistically significant at 5% level; ***-Not statistically significant.

Table 5: second part GLM model (Poisson distribution of errors and log link)

Indicator	Variable	Coefficient	Z
Other	Constant	6.468201*	30.44
Distance	Distance(i,j) Distance(i,j)*distance(i,j)	-0.0423718* .0000776*	-16.94 13.95
Perceptions	Population(i)*gravity accessibility index(i)	2.95e-07*	15.73
Need and socio-economic	Population(i)*demographic need index(i)*Illiteracy rate(i) Population(i)*population(i)	.0000354* -2.97e-12*	4.14 -.48
Geographic variations	Dummy population in north region(i) Dummy population in centre region(i)	-.2022617** -.3838279*	2.10 -3.37
Primary care	Primary care utilization(i)/population(i)	-.1160661*	-2.10
Supply availability	Discharges(j)	.0000352*	6.56
Alternative supply	Hospital competition "index" (i)	-.1873067*	-3.57
Institutional factors	Dummy for first hospital(i,j)*discharges(j) Dummy for second hospital(i,j)*discharges(j) Dummy for central hospital(i,j) Dummy for population in the north using the Porto hospital site(i,j)*discharges(j) Dummy for population in the centre using the Coimbra hospital site(i,j)*discharges(j) Dummy for population in the south using the Lisbon hospital site(i,j)*discharges(i)	0000231* .0000141* -4.304794* .0000158* .0000255* -.0000136*	6.54 4.3 -4.30 2.70 2.99 -5.79

Model summary: 2217 observations; LogLikelihood=-297950

Statistically significant at 1% level; ** Statistically significant at 5% level.

28 Interaction terms against distance were found significant in previous studies (Gesler and Meade 1988). Interaction terms are easily interpretable.

29 For example, private supply is related with higher populated areas and with socio-economic level. In this application, hospital size was almost perfectly related with the number of specialties available on the site.

30 Robust estimates of the errors variance have been used (Huber-White estimates of the variance-covariance matrix).

Most results are as expected. Important findings are[31]: utilisation flows are negatively influenced by alternative hospital supply for the population area; supply availability and institutional factors such as the classification of hospitals of reference (captured by variables of closest and second closest hospitals that represent the hospitals of reference) play a key role in the flows; and primary care utilisation is acting as a substitutive of public hospital care. One finding requires explanation is the different values reported for the central hospitals.

These results imply that the behavioural constraint to be included in the MP model assumes the format:

$$\log U^{'}_{i,j} = others_{i,j} + \alpha_0 * D_j + \alpha_1 * \tilde{D}_i + \alpha_2 * DumFirst_{i,j} * D_j + \alpha_3 * DumSecond_{i,j} * D_j + \alpha_4 * DumPorto_{i,j} * D_j + \alpha_5 * DumCoimbra_{i,j} * D_j + \alpha_6 * DumLisboa_{i,j} * D_j \quad (9)$$

with: $DumFirst_{i,j}$ dummy for the closest hospital of a population point; $DumSecond_{i,j}$ as the dummy for the second closest hospital of a population point; $DumLisboa_{i,j}$, $DumPorto_{i,j}$ and $DumCoimbra_{i,j}$ as the dummies for the central hospitals sites in Lisbon, Porto and Coimbra from population from the South, North and Centre respectively; and $others_{i,j}$ capturing the influence of all the other factors *(ceteris paribus)*.

Mathematical program results

The mathematical program was run inside the AIMMS software package (version 3.1) (Paragon Decision Technology 2000). Results are summarized in tables 6-8. One lower bound was set for hospital size variations of 80% of the current size, and two upper bounds were set for 120 and 200% of the current size.

The model redistributes supply to improve equity (set at locational accessibility and distance) but reduces utilisation (table 6). As the model does not impose any restrictions to variations in flows, there is no improvement on the variability of utilisation rates per population area. Results were found to be very robust to alternative specifications of the behavioural constraint (on the second part of the econometric model).

Table 6: variations in distance and utilisation levels (20% maximum decrease and 20/100% maximum increase in hospital sizes; closest hospital model in the objective function)

	Status quo utilization (predicted)	Utilisation (20% max increase)	Utilisation (100% max. increase		Status quo distance	Distance (20% max.increase)	Distance (100% max. increase)
Total	923455	675306	659412	Average per patient	16	14.4	14.2
Max utiliz per capita	379%	327%	365%	Max distance	88	92	92
Min utiliz per capita	18%	12%	12%	Min distance	0	2	2
Stand. deviation	0.52	0.56	0.63	Stand. deviation	17.5	17.0	16.9

The main findings about redistribution are:

[31] Empirical findings are mainly based on the second part of the model. Other empirical findings relevant for policy analysis are: the impact of determinants of the probability of the flow and the determinants of the level of utilisation differ (this is as expected from previous applications (Pohlmeier and Ulrich 1994)); distance is a deterrent for hospital utilisation; utilisation increases with need factors; perceptions on accessibility have a positive impact on utilisation; and there are geographic variations on the level of utilisation flows across region health system areas that might also be interpreted as non-controlled factors (for example, variations in health policies or in welfare systems). It is outside the scope of this study to develop the analysis on the outputs given by the two-part model.

a) Redistribution discriminates positively towards[32]: hospitals in rural areas with low geographic accessibility, in the south and interior of the country; peripheral hospitals inside urban areas; and smaller hospitals.
b) Supply tends to be moved to 5 health districts (table 7, column A vs. B) –Aveiro, Beja, Braga, Faro and Vila Real; the north is the most undersupplied health region (table 8);
c) For some districts there is a conflict between improving equity in utilisation and progressing to a supply distribution in accordance to need for hospital care (column A vs. B vs. D): that is the case for Viana do Castelo in which current supply is above the target based on relative needs, but for which redistribution based on utilisation suggests a level supply below the needs target;
d) When it is used 20% of maximum variation in hospital size, there are 59 winners against 9 losers (table 8);
e) If no upper bounds are used, the model tends to concentrate supply in a small number of hospitals;
f) Sensitivity analysis on the impact of different equity target points on district redistribution shows a high level of stability of estimates (table 7: column B vs. C).

Table 7: redistribution of supply at the health district level

District	Current supply (A)	Redistribution (20% maximum variation and closest hospital target) (B)	Redistribution (20% maximum variation and three closest hospitals target) (C)	Supply distribution based on estimates of relative needs[33] (D)
Aveiro	50996	61195	56054	68890
Beja	11747	14096	14096	15309
Braga	68102	77342	71467	79415
Braganca	16655	19986	19986	15309
Cast. Branco	22500	22082	22082	20093
Coimbra	88533	74570	74570	42100
Evora	13025	15630	15630	17223
Faro	29717	35660	35660	35402
Guarda	14159	16991	16991	18179
Leiria	35370	37471	34711	44013
Lisboa	250613	212638	212638	207627
Portalegre	10971	10238	10238	12439
Porto	177028	171721	187746	172225
Santarem	35104	34666	34526	44013
Setubal	63925	76710	76710	74631
V. do Castelo	25390	23226	23226	24877
Vila Real	19355	30468	28361	22963
Viseu	23618	22116	22116	40186

Table 8: number of winner sites from redistribution (20% maximum decrease and 20 or 100% maximum increase in hospital site supply)

"Entities"	Winners (20% maximum increase)	Winners (100% maximum increase)
3 health regions	North winner ; Centre maintain; South loser	North winner ; Centre loser; South winner
18 districts	10 winners	8 winners
68 hospital sites	59 winners	34 winners

Discussion

This section: summarises the methodological achievements; discusses the empirical results; and refers some possible developments to the proposed model.

[32] More outputs from the model can be requested to the author.
[33] Based on author estimates: population numbers corrected by age-sex cost curve and by age specific mortality rates.

A new approach to the problem of redistributing hospital supply was reported. The concept of flow-demand has shown to be useful in capturing hospital interactions. The behavioural model is an improvement on previous studies that have excluded variables that influence hospital utilisation. The two-part model was found adequate for taking into account the mixed nature of utilisation data. The mathematical programming application to the Portuguese health care system has generated robust results to alternative econometric formulations; and generated estimates for changes on hospital supply in accordance with the objective of improving equity of utilisation.

The results indicate how the current inequities in utilisation can be reduced by changing supply. Some issues arise:

- The loss in the total amount of utilisation in the system can be interpreted as an equity price for the improvement of accessibilities. This relates to a early consequence of the use of a non-linear relationship (logarithm) between hospital size and flows, which implies that the impact of a decrease in size of big hospitals on utilisation more than offsets the gains in utilisation generated by an increase in size of small hospitals (vide footnote 17).
- Several econometric specifications were produced for capturing specificities of the largest central hospitals, and were used inside the MP model. Results were robust to these variants. Further research seems to be useful on the role that each central hospital is playing in the system. In addition, it is found that any gains in equity of utilisation will depend on a decrease on the level of centralisation in the system.
- About the finding that the model tends to concentrate supply in a reduced number of hospitals, it must be read that hospital supply (by itself) might be a weak instrument for improving equity of access. Any policy that attempts to improve equity on hospital utilisation will have to make use of other health sector and non-health sector policies.
- The econometric application would be improved if there were more data on geographic flows, such as data on primary care utilisation flows, on private hospital utilisation and on other variables of the welfare system. This would help to overcome multicollinearity problems.
- Discharges were assumed to be a good proxy for size, which implies a neglect from some variations in quality, costs, efficiency variations, heterogeneity of capital, etc.

The proposed method suffers from some weaknesses. First, dynamic changes on local social geography and institutional behaviour in the health sector may shift (McLafferty 1988), which might imply problems in using the behavioural model for prediction. Second, there was no account for the financial consequences of redistribution. Third, it was assumed a constant substitutability between hospital alternatives across all the territory, which is a restrictive assumption. Lastly, this is a general model for hospital services that does not provide useful information for analysis at the specialties level.

Further work could consider: the development of better proxies for hospital size and for the capacity of changing production levels; research on the financial and managerial consequences of redistributing supply; examination of the potential interaction between primary and hospital care; sensitivity analysis on other equity indexes; and research on improving the modelling of hospital interaction.

Some issues were left outside the model but require for future attention: testing for simultaneity between need and supply[34] or for spatial autocorrelation (Anselin 1988); and changing the assumption of fixed probabilities inside the MP model.

[34] Though, we are studying flows and not utilization rates, the simultaneity hypothesis between need and supply is not so likely to operate for the Portuguese system.

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