

# Childhood Health, Educational Attainment and Occupational Choice under Risk

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December, 2007

## Abstract

*Aims:* The correlation between health and educational attainment has been the subject of much theoretical and empirical investigation. This paper contributes to the existing literature by developing a mathematical model to examine the effects of uncertainty on the relation between child health and schooling, and child health and occupational choice.

*Methods:* We construct a fisherian two period model in which future labor earnings are randomly dependent on current investments in health and schooling. This framework is used to examine the relations between child health and schooling, and child health and occupational choice in an environment of credit and insurance market failures.

*Results:* We find that when credit markets are perfect and the rate of return on household savings is fixed, uncertainty in the returns to human capital will lead to a positive correlation between child health endowments and schooling attainment. This positive correlation between child health and schooling will be exacerbated when credit markets are imperfect. Thus, health inequalities will result in educational inequalities in an environment of uninsured risk. We extend the model to include occupational choice as a jointly determined variable along with health, and show that children with low health endowments will be involved in more hazardous occupations under the conditions of uninsured risk. Lastly, we are able to show that when perfect insurance markets are present, investments in child health, schooling and occupational hazards will be optimal.

*Conclusions:* From the policy perspective, the results of our model argue for the development of insurance markets. Moreover, they suggest that policy interventions that target higher levels of educational investment among the population need to account for the effect of early childhood differences in health.

# 1 Introduction

Education and health are two important components of human capital. In a recent study published in the *Lancet*, Grantham-McGregor et al (2007) estimate that more than 200 million children under 5 years of age fail to reach their potential in cognitive development because of poverty, poor health and nutrition, and deficient care.

Grossman and Kaestner (1997) review a number of studies which find a highly positive relationship between health and education. However, models investigating pathways through which health and education interact typically assume a non-stochastic setting. This is an unrealistic assumption. It is widely recognized that human capital is subject to considerable risk (Becker 1964, Schultz 1971, Thurow 1970). In the presence of uninsured risk, human capital investment will not be optimal.

In this paper, we will analyze the effect of credit and insurance market failures on household investments in health and education. Market failures are a pervasive feature in developing countries and many economists have argued that uninsured risk is a cause rather than just being an aspect of poverty. It is important for studies on health and education to recognize the impact of risk. This paper develops a model of household investment in child health and education under risk. This allows us to examine the effect of uninsured risk on the causality from health to education and vice versa. Our model allows us to make many empirically relevant observations on the correlation between health and schooling under risk. The paper is organized as follows. In section 2 of the paper, we will review the literature on education and health linkages, and on the impact of risk on investments in education and health. In section 3, we will outline our model. In section 4, we will analyze the effect of uninsured risk on investments in health and education assuming the presence and absence of credit markets. In section 5, we will analyze the level of household investment in health and education in the presence of perfect insurance markets. In section 6, we will consider an extension to our model. In section 7, we will conclude.

# 2 Literature Review

Becker, in his pioneering work, drew an analogy between investment in health capital and investment in other forms of human capital such as education. His model was developed by Grossman (1972), who constructed a model of the demand for health. In Grossman's (1972) model, individuals demand health for two reasons. As a consumption commodity, health enters directly into their utility functions. And, as an investment commodity, health determines the total amount of time available for work in the market sector of the economy where consumers produce money earnings, and for work in the non-market or household sector where they produce commodities that enter their utility function.

Grossman's (1972) model has been extended and analyzed in non-stochastic and stochastic settings. For instance, Edwards and Grossman (1977) develop a theoretical model to look at the interrelationships between various aspects of children's health and their intellectual development. Their model suggests that for understanding the behaviour of parents with respect to their children's health and development it is important to distinguish low income families from high income families. A similar result is obtained in an overlapping generations model developed by Galor and Mayer-Foulkes (2002). The authors focus on the effects of minimal health requirements for acquiring an education. They show that when families do not have enough resources to invest in the satisfaction of basic needs and health care, and finance is not available for this purpose, a poverty trap may exist with low health, education and income. This approach adds to the literature that explains the persistence of poverty through the presence of credit constraints. Other studies have also looked at the effect of insurance market failures and uninsured risk on a household's decision to invest in education or health. Dardanoni and Wagstaff (1990) introduce uncertainty into the pure consumption version and Cropper (1977), Murrinen (1982), Dardanoni and Wagstaff (1987), Selden (1993), Chang (1996) introduce uncertainty into the pure investment version of Grossman's model of demand for health. Liljas (1998) introduces uncertainty in a multi-period mixed investment-consumption version of Grossman's model of demand for health. Levhari and Weiss (1976) is a seminal paper that introduces uncertainty into a model of investment in education and shows that investment in education will be positively correlated with the level of a household's initial income.

On the empirical side, a vast literature reviewed in Grossman and Kaestner (1997) finds a positive and significant correlation between health and education in a non-stochastic setting. This correlation is robust even after controlling for different measures of socio-economic status, such as income and race, and regardless of whether health levels are measured by mortality rates, self-reported health status, or physiological indicators of health. Alderman et al (2001) find that child health (nutrition) is three times as important for enrollment than suggested by naive estimates that assume that child health is predetermined rather than determined by household choices in the presence of unobserved factors such as preferences and health endowments. Mayer-Foulkes (2003) provides evidence from Mexico that childhood nutrition and health as well as parental education have substantial and possibly increasing returns in the acquisition of education as measured by school permanence. The poor are less able to invest in human capital and constituent elements for a low human capital trap or for a prolonged transition in intergenerational human capital accumulation are present in Mexico.

In an interesting recent paper, Gan and Gong (2007) have estimated the interdependence between health and schooling using a stochastic dynamic programming model. They investigate the extent to which and the channels through which health and educational attainment are interdependent. The estimation results strongly support the interdependence between health and education. In particular, the estimated model indicates that an individual's education,

health expenditure and previous health status all affect his current health status. Moreover, the individual's health status affects his mortality rate, wage, home production and academic success. However, Gan and Gong (2007) do not differentiate exactly how much of the estimated effect of health on education and vica versa is being driven by credit as opposed to insurance market failures. A similar situation is obtained in a seminal paper by Jacoby and Skoufius (1997) which estimates the effect of financial market imperfections on human capital investment in the context of a developing country. The paper explores the link between financial market incompleteness and human capital accumulation. It examines how child school attendance responds to seasonal fluctuations in the income of agrarian households using panel data from rural India, and finds that seasonal fluctuations in school attendance are a form of self-insurance, that is, households withdraw children from school when they experience a negative income shock. However, again the paper is unable to differentiate between the effects of credit and insurance market failures. As we shall see below, our model is able to differentiate the effect of credit and insurance market failures on the correlation from health to schooling.

### 3 The Model

Consider a household with one adult and one child. The household decision making process is unitary with the adult making the decision to invest in the child's education and health. There are two time periods, the first (present) and the second (future). In the first period, adult ( $Y$ ) and child wages ( $W$ ) are the two sources of household income. In the second period, income accrues from the interest earned on household savings in the first period and as a function of the child's human capital (education and health) accumulated in the first period. The two period utility function is represented by  $U(C_1, C_2)$ , where  $C_1$  denotes consumption in the first period and  $C_2$  denotes consumption in the second period. Investments in health and education are time consuming.  $A$  is the proportion of the first period time devoted to schooling.  $M$  is the proportion of the first period time devoted to health. We assume that the opportunity cost of child's time is the only cost associated with investments in health and schooling. There are no direct costs of schooling and health. Investment in human capital in the second period is identically zero. We ignore leisure - both of the child and of the adult. Future earnings  $\pi(\cdot)$  depend on current investments in health and education and on the future (unknown) state of the world  $\theta$  where  $\theta$  is a random variable with a known distribution. This is illustrated as follows:

$$U(C_1, C_2) = U(C_1) + \beta EV(C_2)$$

$U(C_1, C_2)$  is assumed to be atleast three times continuously differentiable and to possess everywhere positive marginal utilities. Thus,

$$U'(C) > 0, U''(C) < 0, U'''(C) > 0$$

$C_2$  is the consumption in the second period and is defined as;

$$C_2 = (Y + W.(1 - A - M) - C_1).(1 + r) + \pi(A, H_0, \theta)$$

where  $H_0$  is the endowment of health that the child is born with in period one.

## 4 Absence and Presence of Capital Market Imperfections

### 4.1 Cases in the presence of a perfect capital market

Assume the existence of perfect capital markets and a fixed rate of return on savings. Also, assume that insurance markets are missing so that investment in education is risky and the risk is uninsured. Thus, in the first period the household has the choice of investing in (risk free) financial assets or (risky) education. This model is a variant of Arrow's model of portfolio selection (Arrow, 1965). We show (in the first proposition below) that under these conditions there will exist a positive correlation between childhood health and educational attainment. Those with poor health will acquire less education and vice versa.

The household maximization problem is as follows:

$$\text{Max}_{C_1, A} U(C_1) + \beta EV((Y + W(1 - A) - C_1).(1 + r) + \pi(A, H_0, \tilde{\theta}))$$

The first order conditions are:

$$U'(C_1) - \beta EV'(C_2).(1 + r) = 0$$

$$-W\beta(1 + r)EV'(C_2) + \beta\pi_A EV'(C_2\tilde{\theta}) = 0$$

Given this maximization problem, we can make an interesting observation. The interesting observation arises from looking at the way in which risk enters the second period income generating function. We can show below that if the risk is multiplicative, the household's optimal investment in education under risk will be less than under certainty. The converse will hold true when the risk is additive. This observation is made following Levhari and Weiss (1976).

Rewriting the second of the two first order conditions above, we get the following,

$$\beta\pi_A EV'(C_2\theta) = W\beta(1+r)EV'(C_2)$$

From the definition of covariance and rearranging the above expression, we get the following,

$$E(\pi_A\theta) = W(1+r) - \frac{Cov(v', \pi_A\theta)}{EV'}$$

where  $Cov(v', \pi_A\theta) < 0$

as  $\frac{d^2v(A, \theta)}{dAd\theta} > 0$

(  $v'$  varies negatively with  $\theta$ , from which we infer the following:

$$\frac{d^2v(A, \theta)}{dAd\theta} < (>)0 \text{ for all } (A, \theta)$$

implies  $cov(v', \pi_A) > (<)0$  )

This implies that the household will invest in education such that the expected returns to education are pushed above the alternative cost of education. In other words, the optimal level of investment in education by the household is less than when there is no risk associated with the income generating function. When the risk is additive, then  $Cov(v', \pi_A\theta) < 0$  and this would lead to a greater investment in education than under the certainty case.

**Proposition 1** *Under the assumption of decreasing absolute risk averse preferences, an increase in health endowment will encourage investment in education when there is risk associated with the second period income generating function.*

**Proof.**

$$Max_{C_1, A} U(C_1) + \beta EV((Y + W(1 - A) - C_1).(1 + r) + \pi(A, H_0)\tilde{\theta})$$

■

The first order conditions are:

$$U'(C_1) - \beta EV'(C_2).(1 + r) = 0$$

$$-W\beta(1+r)EV'(C_2) + \beta\pi_A EV'(C_2\tilde{\theta}) = 0$$

Totally differentiating the first order conditions with respect to  $C_1, A, H_0$  we get:

$$\begin{aligned} & [U''(C_1) + \beta(1+r)^2 EV''(C_2)] .dC_1 + \\ & [\beta(1+r)^2 W^2 EV''(C_2) - \beta(1+r)\pi_A EV''(C_2\theta)] .dA + \\ & [-\beta(1+r)\pi_{H_0} EV''(C_2\theta)] .dH_0 = 0 \end{aligned}$$

$$\begin{aligned} & [\beta(1+r)^2 W .EV''(C_2) - \beta\pi_A(1+r) EV''(C_2\theta)] .dC_1 + \\ & [\beta(1+r)^2 W^2 EV''(C_2) + \beta\pi_A^2 EV''(C_2\theta) - 2\beta(1+r)W\pi_A EV''(C_2\theta)] .dA + \\ & [-\beta(1+r)W\pi_{H_0} EV''(C_2\theta) + \beta\pi_A\pi_{H_0} EV''(C_2\theta)] .dH_0 = 0 \end{aligned}$$

Writing the above two equations in a matrix form, we get the following,

$$\begin{pmatrix} a & b \\ d & e \end{pmatrix} \cdot \begin{pmatrix} dC_1 \\ dA \end{pmatrix} = \begin{pmatrix} c \\ f \end{pmatrix} .dH_0$$

$$\begin{pmatrix} dC_1 \\ dA \end{pmatrix} = \frac{1}{(ae - bd)} \cdot \begin{pmatrix} e & -d \\ -b & a \end{pmatrix} \cdot \begin{pmatrix} c \\ f \end{pmatrix} .dH_0$$

where,

$$\begin{aligned} a & \text{ is } U''(C_1) + \beta(1+r)^2 EV''(C_2) \\ b & \text{ is } \beta(1+r)^2 W .EV''(C_2) - \beta\pi_A(1+r) EV''(C_2\theta) \\ c & \text{ is } -\beta(1+r)\pi_{H_0} EV''(C_2\theta) \\ d & \text{ is } \beta(1+r)^2 W .EV''(C_2) - \beta\pi_A(1+r) EV''(C_2\theta) \\ e & \text{ is } \beta(1+r)^2 W^2 EV''(C_2) + \beta\pi_A^2 EV''(C_2\theta) - 2\beta(1+r)W\pi_A EV''(C_2\theta) \\ f & \text{ is } -\beta(1+r)W\pi_{H_0} EV''(C_2\theta) + \beta\pi_A\pi_{H_0} EV''(C_2\theta) \end{aligned}$$

The second order conditions for the local maximum are that,

$$H = \begin{vmatrix} a & b \\ d & e \end{vmatrix} > 0$$

and,

$$\begin{aligned} a & < 0 \\ e & < 0 \end{aligned}$$

Alternatively,

$$H = ae - bd = ae - b^2 > 0$$

as  $b = d$

Also,

$$\text{sign } \frac{dA}{dH_0} = \text{sign } (-bc + af)$$

implies,

$$-bc + af = \beta^2(1+r)^2 EV''(C_2\theta)\pi_{H_0}(W(1+r)EV''(C_2) - \pi_A EV''(C_2\theta)) + (U''(C_1) + \beta(1+r)^2 EV''(C_2)).(-\beta(1+r)W\pi_{H_0}EV''(C_2\theta) + \beta\pi_A\pi_{H_0}EV''(C_2\theta))$$

where,

Simplifying, we get,

$$-bc + af = -\beta(1+r)U''(C_1)W\pi_{H_0}EV''(C_2\theta) + \beta\pi_A\pi_{H_0}EV''(C_2\theta)U''(C_1)$$

$$\text{sign } \frac{dA}{dH_0} = \text{sign } \beta\pi_{H_0}(U''(C_1).(-W(1+r) + \pi_A)EV''(C_2\theta))$$

Let  $R(C_2) = -\frac{V''(C_2)}{V'(C_2)}$  be the absolute risk aversion of utility function  $V(C_2)$ . Following Arrow et al, we assume that the utility function  $V(C)$  exhibits decreasing absolute risk aversion, that is,  $R'(\cdot) < 0$ .

Let the value of absolute risk aversion which clears the first order conditions under risk be  $\tilde{R}$ . Similarly, let the value of absolute risk aversion which clears the first order conditions in the absence of risk be denoted  $R$ . By the assumption of decreasing absolute risk aversion,  $R$  is monotonically decreasing in consumption and the risk parameter  $\theta$ . This together with the first order conditions gives us:

$$R > (< \tilde{R}) \iff (-W(1+r) + \pi_A)\theta < (> 0)$$

This implies the following,

$$(\tilde{R} - R).(-W(1+r) + \pi_A)\theta < 0$$

$$V''(-W(1+r) + \pi_A)\theta > -V'\tilde{R}(-W(1+r) + \pi_A)\theta$$

Taking expectations on both sides of the above expression, we get,

$$E(V''(-W(1+r) + \pi_A)\theta) > -\tilde{R}E(V'(-W(1+r) + \pi_A)\theta)$$

We can note from the first order conditions, that the right hand side of the above expression is zero. Thus, the above expression can also be written as;

$$E(V''(-W(1+r) + \pi_A)\theta) > 0$$

The above expression together with our expression derived earlier for the sign of  $\frac{dA}{dH_0}$  ( $\text{sign } \frac{dA}{dH_0} = \text{sign } d - e = \text{sign } (\beta EV''\pi_{H_0}(-W(1+r) + \pi_A)\theta)$ ) gives us that;

$$\frac{dA}{dH_0} > 0$$

Note, that in deriving the above result we made the assumption that the household has decreasing absolute risk averse preferences. Another implicit assumption, given multiplicative risk, is that there is increasing risk. Thus, with an increase in household investment in the risky asset household exposure to risk increases. In other words,  $\frac{d\pi}{d\theta}$  measures the sensitivity of the realized wage to the shock  $\theta$ , the cross partial measures how this sensitivity varies with the level of education  $A$ . If the cross partial derivative  $\frac{d^2\pi(A, \theta)}{dAd\theta}$  is positive, then human capital makes the realized wage more sensitive to the shock  $\theta$  and thereby exposes the agent to more wage risk.

This result shows that an increase in health endowment will encourage investment in education. Children born with better first period level of health will receive more investment in their education as compared to children born with poor levels of health. Interestingly, it can also be observed that when the rate of interest is variable, then an increase in rate of interest could possibly invert the sign of the relation between investment in education and the initial health endowment from positive to negative.

#### 4.1.1 Model of investment in 3 assets: health, schooling and savings and Complementarity in health and schooling investments

Let's now assume that education and health are complementary inputs in the income generating function. We extend our model in the following manner. There is a health production function  $I(\cdot)$  and there is a education production function  $Q(\cdot)$ . We assume that initial/childhood health enters as an input into the second period educational attainment function by increasing the productivity of first period investment in education. Our assumptions  $Q_{H_0} > 0$  and  $I_{D_0} > 0$  are akin to Heckman's assumptions of universal self productivity or recursive productivity which in the context of our model imply that health endowment and schooling ability are inputs into the production of schooling and health respectively. This formulation is sufficiently general to allow cross effects of initial endowments of health or schooling ability on investments into schooling and health respectively. In other words, the concept of universal direct complementarity of investments is that  $\frac{\partial^2 Q}{\partial A \partial H_0} > 0$ . This means that higher levels of initial health increase the productivity of investment in education.

Accordingly, we define  $C_2$  as the following:

$$C_2 = (Y + W \cdot (1 - A - M) - C_1) \cdot (1 + r) + \pi(D, H, \theta)$$

where,

$$H = H_0 + I(M, H_0, D_0)$$

$$D = D_0 + Q(A, H_0, D_0)$$

$H_0$  is the endowment of health that the child is born with in period one.  $I$  is the technology that converts the investment in child health in the first period into net additions to health stock over the two period time framework. Similarly,  $D_0$  is the schooling ability or endowment that the child is born with in period one.  $Q$  is the technology that converts the investment in child schooling in the first period into net additions to education stock over the two periods.

**Proposition 2** *If the level of initial health enters as an input into the second period level of education and if initial schooling ability enters as an input into the second period health, that is, if health and education are complements, then there will be a positive correlation between the level of investment in education and the initial level of health in the absence of risk. In the presence of risk, this positive correlation will get exacerbated.*

**Proof.** If we extend our model to allow for the complementarity between health and schooling investments, we get the following structure, ■

$$\text{Max}_{C_1, A, M} U(C_1) + \beta EV(C_2)$$

$$\text{where } C_2 = (Y + W.(1 - A - M) - C_1).(1 + r) + \pi(D, H, \theta)$$

$$H = H_0 + I(M, H_0, D_0)$$

$$D = D_0 + Q(A, H_0, D_0)$$

The FOCs are as follows:

$$U'(C_1) - \beta EV'(.).(1 + r) = 0$$

$$-\beta.(1 + r).EV'(.). + \beta E(V'\theta).\pi_D Q_A = 0$$

$$-\beta.(1 + r).EV'(.). + \beta E(V'\theta).\pi_H I_M = 0$$

(proof as for the previous proposition) Even in the absence of risk  $\frac{dA}{dH_0} > 0$ .

In the presence of risk, this positive correlation between initial/childhood health and educational investment will be exacerbated.

#### 4.1.2 Alternative technological assumptions for income generating function

**Proposition 3** *If the returns to education are certain and seperable from the returns to health, then the optimal level of investment in education will not vary with the level of exogenous health endowment, when there is risk associated with the stock of health.*

**Proof.**  $Max_{C_1, A} U(C_1) + \beta EV((Y + W(1 - A) - C_1) \cdot (1 + r) + f(A) + \pi(H_0)\theta)$

■

The first order conditions are as follows,

$$U'(C_1) - \beta EV'(C_2) \cdot (1 + r) = 0$$

$$-W\beta(1 + r)EV'(C_2) + \beta f' EV'(C_2) = 0$$

From the definition of covarince and rewriting second of the two first order conditions above, we get,

$$E(f') = W(1 + r) - \frac{Cov(v', f')}{EV'}$$

$$\text{where } Cov(v', f') = 0$$

as the returns to educational investment are assumed to be constant. In this framework, the level of risk affects the household consumption decisions but household production decisions are left unaffected. The optimal level of educational investment will be given by equating the returns to education to the alternative cost of educational investment. Also, it can be noticed that the optimal level of education is not a function of the level of the level of initial health endowment. It is immediately seen from the first order condition that  $\frac{f'}{W} = 1 + r$ . for all states of the world. Uncertainty in this additive formulation has no effect on the production decisions of the individual, only the consumption decisions are affected.

Formally, it can be proven (*in a similar manner to the proposition 1*) that,  $\frac{dA}{dH_0} = 0$ .

**Proposition 4** *If education and health enter as additive components of the income generating function in the second period, then risk associated with the exogenous health endowment, will cause the level of investment in education to decline with an increase in health endowment.*

**Proof.**  $Max_{C_1, A} U(C_1) + \beta EV((Y + W(1 - A) - C_1).(1 + r) + \pi(f(A) + H_0\theta))$

■

The first order conditions are as follows,

$$U'(C_1) - \beta EV'(C_2).(1 + r) = 0$$

$$-W\beta(1 + r)EV'(C_2) + \beta\pi_A f' EV'(C_2) = 0$$

Rewriting second of the two first order conditions above, we get,

$$E(\pi_A f') = W(1 + r) - \frac{Cov(v', \pi_A f')}{EV'}$$

where  $Cov(v', \pi_A f') > 0$

The covariance term in the above equation is positive, so that risk averse individuals push the expected rate of return below the safe rate, increasing their investment in education. Intuitively, individuals spend more on education than in the certainty case, because individuals experience a high marginal utility of consumption in precisely the same states of nature as those in which there are high marginal returns to education. This positive relation makes educational investment relatively more valuable than saving, which has a safe but uncorrelated return. The results above plus an assumption of declining absolute risk aversion lead us to the result that  $\frac{dA}{dH_0} < 0$ . Those poor in health will consume more education than those rich in health (assuming similar initial levels of schooling ability). Intuitively, it is risky for those poor in health in this framework not to invest on education.

## 4.2 Cases under investment imperfect capital markets

### 4.2.1 Model of investment in 2 assets: health and schooling

Suppose, the capital market is imperfect. Thus, there are liquidity constraints and a household can invest only in health and in education. In other words, the household has zero savings. In such a case, the following propositions can be proven.

**Proposition 5** *If the capital market is imperfect so that a household can either invest in schooling or health but can not save, then a risk associated with the second period income generating function will induce a positive correlation between initial or childhood health and subsequent investment in education.*

**Proof.**  $Max_{A,M} U(Y + W(1 - A - M)) + \beta EV(\pi(A, H)\theta)$  ■

$$H = H_0 + I(M, H_0)$$

The first order conditions are given as follows,

$$\begin{aligned} -W.U'(C_1) + \beta EV'(C_2\theta)\pi_A &= 0 \\ -W.U'(C_1) + \beta EV'(C_2\theta)\pi_H I_M &= 0 \end{aligned}$$

Totally differentiating the FOCs wrt  $A, M$  and  $H_0$ , we get,

$$\begin{aligned} [W^2 U''(C_1) + \beta EV''(C_2\theta)\pi_A^2] .dA + [W^2 U''(C_1) + \beta EV''(C_2\theta)\pi_A \pi_H I_M] .dM + \\ [\beta EV''(C_2\theta)\pi_A \pi_H (1 + I_{H_0})] .dH_0 = 0 \end{aligned}$$

$$\begin{aligned} [W^2 U''(C_1) + \beta EV''(C_2\theta)\pi_H I_M \pi_A] .dA + [W^2 U''(C_1) + \beta EV''(C_2\theta)\pi_H^2 I_M^2] .dM + \\ [\beta EV''(C_2\theta)\pi_H^2 I_M (1 + I_{H_0})] .dH_0 = 0 \end{aligned}$$

Rewriting the above equations in matrix notation we get,

$$\begin{pmatrix} a & b \\ d & e \end{pmatrix} \cdot \begin{pmatrix} dA \\ dM \end{pmatrix} = \begin{pmatrix} c \\ f \end{pmatrix} .dH_0$$

From the first order conditions, we get the optimality condition that  $\pi_A = \pi_H I_M$ . Substituting this in the  $f$  term above, we can see that  $c$  is equal to  $f$ .

Also,

$$\begin{pmatrix} dA \\ dM \end{pmatrix} = \frac{1}{H} \times \begin{pmatrix} e & -d \\ -b & a \end{pmatrix} \times \begin{pmatrix} c \\ f \end{pmatrix} \times dH_0$$

By the second order conditions for a local optima, we get that  $H > 0$

From Arrow's definition of absolute risk aversion:

$$R = \frac{-V''}{V'}$$

with  $R' < 0$  for all  $C_2 \geq 0$  (via the assumption of declining absolute risk aversion)

Let the value of  $R$  which clears the first order conditions under risk be denoted  $\tilde{R}$ . Similarly, let the value of  $R$  which clears the first order conditions in the absence of risk be denoted  $R$ .  $R$  is monotonically decreasing in consumption and the risk parameter  $\theta$ . This together with the first order conditions gives us:

$$R > (< \tilde{R}) \iff (\pi_A - \pi_H I_M)\theta < (> 0)$$

which implies,

$$(\tilde{R} - R) \cdot (\pi_A - \pi_H I_M) \theta < 0$$

$$V''(\pi_A - \pi_H I_M) \theta > -V' \tilde{R}(\pi_A - \pi_H I_M) \theta$$

Taking expectations;

$$E(V''(\pi_A - \pi_H I_M) \theta) > -\tilde{R} E(V'(\pi_A - \pi_H I_M) \theta)$$

RHS of the above inequality is zero from the first order conditions. Thus,

$$E(V''(\pi_A - \pi_H I_M) \theta) > 0$$

This together with our equation derived before for the sign of  $\frac{dA}{dH_0}$  (sign  $\frac{dA}{dH_0} = \text{sign } d - e = \text{sign } (\beta E V'' \pi_H I_M (\pi_A - \pi_H \pi_M) \theta)$ ), gives us that:

$$\frac{dA}{dH_0} > 0$$

Similarly,  $\frac{dM}{dH_0} > 0$

In this framework, those with poor health will invest less in education, as well as less in health. One could compare the magnitude of this effect with that derived earlier.

## 5 Optimal investment in the presence of perfect insurance markets

**Proposition 6** *If the household has access to competitive insurance markets where they can contract at rate  $\Delta = \theta - 1$  per unit of human capital, returns to human capital are continuous random variables and savings are interior, human capital investment will be optimal.*

**Proof.** To prove this, we first show that expected profit of insurance company is zero. We also derive the condition that given the availability of an insurance contract, a household will maximize subject to the same constraints as it would were there no risk. ■

The profit  $\Pi$  of the insurance company is defined as:

$$\Pi = \theta \pi(A, H_0)$$

Taking expectations,

$$E(\Pi) = E(\theta)\pi(A, H_0) = 0$$

as  $E(\theta) = 1 - E(\Delta) = 1 - 1 = 0$ , and  $\pi(A, H_0)$  is predetermined.

In the following, we show that with the provision of insurance, the constraint in the household's optimization problem is reduced to the one without risk.

Without insurance the second period consumption is;

$$C_2 = \pi(A, H_0)\theta + (Y + W(1 - A) - C_1).(1 + r)$$

With insurance, the second period consumption is,

$$\begin{aligned} C_2 &= \pi(A, H_0)\theta + (Y + W(1 - A) - C_1).(1 + r) - \Delta\pi(A, H_0) \\ &= \pi(A, H_0) + (Y + W(1 - A) - C_1).(1 + r) \end{aligned}$$

which is the consumption without risk. Thus, the human capital investment decisions will be optimal when insurance against risk is available.

## 6 Extending the model

### 6.1 Occupational linkage between health and schooling and the effect of risk

We now extend our model to incorporate occupational hazards as a choice variable within our maximization problem. The occupational linkage can be interpreted on the lines of Kemna (1982). In this formulation, occupational hazards enter into the wage, schooling and health production functions. Kemna brings together insights from labor economics (the theory of compensating wage differentials) and health economics to note that an increase in occupational hazards will increase the wage and decrease the health stock. Furthermore, he empirically estimates his model and shows the existence of a small occupation effect in the schooling-health relationship but this effect is conservatively estimated and biased downward given the presence of unobserved differences in individuals' initial health status. We adapt his approach to our model in the following manner.

**Proposition 7** *When occupational hazards ( $z$ ) is an endogenous or choice variable, children born with a lower level of health endowment will be involved in more hazardous occupations.*

**Proof.**

$$\text{Max}_{C_1, z} U(C_1) + \beta EV((Y + W(z) - C_1).(1 + r) + \pi(H)\tilde{\theta})$$

where  $H = H_0 + I(z)$  ■

Also,

$$W_z > 0$$

and  $I_z < 0$

The first order conditions are:

$$U'(C_1) - \beta EV'(C_2).(1 + r) = 0$$

$$W_z \beta (1 + r) EV'(C_2) + \beta \pi_H I_z EV'(C_2 \tilde{\theta}) = 0$$

The first order condition can also be written as:

$$-\beta \pi_H I_z EV'(C_2 \tilde{\theta}) = W_z \beta (1 + r) EV'(C_2)$$

subtracting  $-\beta \pi_H I_z EV'(C_2)$  from the LHS and RHS, we get,

$$-\beta \pi_H I_z EV'(C_2(\tilde{\theta} - 1)) = (W_z \beta (1 + r) + \beta \pi_H I_z EV'(C_2)).EV'(C_2)$$

subtracting  $-\beta \pi_H I_z E(EV'(C_2(\tilde{\theta} - 1))) = 0$  from the LHS of the above equation, we get

$$-\beta \pi_H I_z Cov(V', \theta) = (W \beta (1 + r) + \beta \pi_H I_z).EV'(C_2)$$

$Cov(V', \theta) < 0$  and since  $-\beta \pi_H I_z > 0$  we have that the LHS of the above equation is negative. And, thus, for the RHS to be negative, we need;

$$W \beta (1 + r) + \beta \pi_H I_z < 0$$

or  $\pi_H I_z < W(1 + r)$

which implies that investment in occupational hazards is inefficiently high.

Assuming decreasing absolute risk aversion, it can be shown in a similar manner to the previously proven propositions that:

$$\frac{dz}{dH_0} < 0$$

## 7 Conclusion

In this paper, we have shown that when credit markets are perfect and the rate of return on household savings is fixed, uncertainty in the returns to human capital will lead to a positive correlation between child health endowments and schooling attainment. This positive correlation between child health and schooling will be exacerbated when credit markets are imperfect. Thus, health inequalities will result in educational inequalities in an environment of uninsured risk. We extend the model to include occupational choice as a jointly determined variable along with health, and show that children with low health endowments will be involved in more hazardous occupations under the conditions of uninsured risk. Lastly, we are able to show that when perfect insurance markets are present, investments in child health, schooling and occupational hazards will be optimal. From the policy perspective, the results of our model argue for the development of insurance markets. Moreover, they suggest that policy interventions that target higher levels of educational investment among the population need to account for the effect of early childhood differences in health.

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