Scale and optimal size of sites in hospitals
Empirical results for a panel of Danish public hospitals

Troels Kristensen[1]

& Kim Rose Olsen[12]

[1]Institute of Public Health, Department of health Economics, Faculty of Social Sciences
University of Southern Denmark
&
[2]Danish Institute for Health Service Research

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Correspondance:

Troels Kristensen

University of Southern Denmark

Institute of Public health, Department of health Economics

Windsløwparken 9B

5000 Odense C.

E-post:trk@sam.sdu.dk

Phone +45-65503906

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Abstract:

Context and aim

The Danish hospital sector is facing major restructuring over the coming decade. There are plans for a significant rebuilding programme, driven by a political desire to concentrate activity in fewer, larger hospitals than at present. The evidence base underpinning these decisions is not apparent. Our study analyses whether there are unexploited economies of scale in the current configuration of Danish hospitals

Methods

We estimate cost functions using stochastic frontier analysis and fixed effect models for all Danish hospital sites, and use the short-run function to derive estimates of long-run scale economies by applying the envelope condition. We estimate economies of scale for the sample as a whole and for sub-groups of hospital and estimate the optimal size of a hospital site. We consider whether results are sensitive to grouping sites within the legal hospital entity to which they belong and functional form. We perform and compare cross-sectional and fixed effects panel data estimates.

Data

We analyse data for all Danish hospital sites (we exclude a handful of specialist sites). We use administrative data on total costs, DRG-casemix weighted activity, number of beds, and other hospital characteristics for three years from 2004.

Results

Overall we identify significant to moderate long run economies of scale, when applying two alternative translog cost function. However using a quadratic functional form we identify constant economies of scale for the medium sized sub-groups and decreasing economies of scale for the largest sub-groups. The optimal number of beds per site is estimated to be either 204 or 275 beds per site, depending on the chosen model.

Conclusions

This is work in progress we would welcome discussion on model specification and our approach to deriving scale economies and optimal size.

Keywords: Economies of scale, optimal size, hospitals, cost function, short run, long run.

1. Introduction

In Denmark the number of somatic hospitals has decreased from 117 in 1980 to 52 in 2004. Some of this decrease is due to the fact that the definition of hospital sites has changed, from in 1980 always to be a hospital at one location, to now being a conglomeration hospital distributed at more satellite hospitals. A quality reform of the Danish government, as well as a strategy and investment plan of the Danish Regions for the future health services show that this development is expected to continue in the years to come, Danish Regions et al. (2007). The Danish government and the 5 regions have plans for a significant rebuilding program including green field investments at 5 new sights, significant extension and reconstruction of several existing hospitals and to merge or close a few small hospitals[1].

Whether the rising concentration of the somatic hospital treatments on fewer hospital unities is an advantage might depend on whether there are economies of scale in connection with the production at the hospitals. Exploitation of economies of scale in the hospitals may help to limit expenses for health outputs without compromising the quality and the amount of health outputs. On the other hand, hospitals can end up in a situation, where the hospitals become so large that the cost of treatment all in all gets too large due to diseconomies of scale. Furthermore, plans to concentrate further, anticipate that the optimal relationship between hospital size in terms of number of beds and activity is quite large.

The evidence base underpinning the rebuilding program decisions is not apparent from an economic perspective. To our knowledge no studies have been conducted in Denmark. Therefore the aim of this study is to asses whether there are unexploited economies of scale in the current configuration and to estimate an "optimal" relationship between hospital size and hospital activity. We use the number of beds per hospital when measuring hospital size as it has become accepted in the literature[2].

This paper is differentiated from former studies since the unit of analysis is hospital sites instead of the entire conglomerate hospital defined by a hospital code. This means that we approximately can interpret the estimated economies of scale and hospital sizes in relation to actual hospital sizes (physical entities). This approach has to our knowledge not been taken before. In the same way a literature review only reviled a few studies which has estimated an optimal hospital size, e.g. Preyra & Pink (2006b). No studies were found which estimated an optimal size of hospital sites.

[1] The association of Danish Regions has published a homepage with the rebuilding program of each region. The address is: www. Godtsygehusbyggeri.dk.

[2] [2]The impact of scale characteristics on the quality of hospital treatments is also central for overall effect of the trend towards concentration of hospitals. The undocumented assumption behind the current changes is that the professional quality will be increased as a function of hospital size.

2. Methods

This study estimates short run and long run cost functions to calculate scale estimates and estimates of optimal size of hospital sites. By econometric assessment of the hypothesis of economies of scale and optimal size of hospital sites a number of choices have to be made.

Unit of analysis

Due to mergers and closures as a result of the structural development in the hospital sector the number of hospitals in Denmark has declined over the last many decades. This means that many hospitals have changed from being a hospital with a single site to conglomerate hospitals with a row of satellite hospitals. When estimating economies of scale and optimal hospital size this is a challenge because the published cost and output data for hospitals have become aggregated at the conglomerate level, which is greater than the physical production entity. Therefore, this study is based on data for each hospital site to be able to reveal policy relevant knowledge about the cost and production characteristics of the physical production entities in the Danish hospital sector.

Model of hospital production and cost

In order to operationalize the cost and the explanatory variable, the costs and hospital output are defined on the basis of the Eurostat definition of hospital inputs intermediate products and outputs. The model interprets hospital outputs as production of finished treatments, Castelli et al. (2007);Smith & Street A. (2007). The remaining production in the hospital is perceived as a multistage production of "intermediate outputs", which are inputs in the production of outputs together with the other inputs as labour, capital and materials. The costs are understood as the cost of "intermediate outputs" and inputs, while hospital outputs are the production of all types of finished treatments.

Models for short run cost functions

To estimate the cost function for the Danish public hospitals three different functional forms have been applied to examine the results sensitivity to the chosen functional forms. The first two specifications are the translog model and the Cobb Douglas model. The third specification is the less common quadratic functional which belongs to the family of “flexible” forms as the translog model.

The three functional forms can be expressed as a version of the family of “flexible” functional forms, which are 2' order's Taylor-approximations to an unknown functional form.

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$$g(C_{it}) = \alpha_0 + \sum_{i=1}^{N} \beta_i f(\mathbf{x_{it}}) + \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij} f(\mathbf{x_{it}}) + u_{it} \qquad t = 1,...,T \qquad (1)$$

where g is a real valued function to the cost C_{it} for hospital i (i =1,2,..,N) in period t (t=1,2…,T), α_0 is a constant, f a real valued function to the cost determinants, x_{it} is a matrix of outputs and costshifters. β is a vector of unknown parameters and u_{it} is a random disturbance term. (1) is equal to the quadratic form when f and g er chosen to be equal to x_{it}. If f and g are chosen to be the natrual logaritmic function (1) yields the translog model. The Cobb douglas version is the special nested case of the translog model where squared and interaction terms in (1) are excluded.

Since this study allows x_{it} to contain control-variables (cost shifters) for hospital i in time period t the estimated models also belong to the family of hybrid cost functions, Granneman (1986).

In connection with the formulation of (1) input prices are left out, as it at the same time is assumed that input prices do not vary between the hospitals. The reason is that pay and agreement conditions are accepted to be fairly uniform across the public hospitals in Denmark, since wages are negotiated nationally and wages mainly vary with the experience and level of education. The public regulation of the hospitals purchases of medicine and other not pay-related inputs means that it is accepted that the hospitals have uniform prices in these areas, even though the purchase departments of hospitals autonomously can negotiate cost prices with certain types of suppliers. Additional arguments for the assumption about constant prices are that it is a panel data study with a relatively short time dimension and that the literature indicate it is difficulty to measure the input prices for hospitals e.g. the price of capital, Aletras (1999);Weaver & Deolalikar (2004).

Specification of cost and output variables

"Flexible" cost functions have more parameters to estimate than the classical Cobb Douglas cost functions. Besides, "flexible" cost functions are exposed to multicollinearity problems which make the choice of covariates an important task (). Thirdly, aggregated output measures from hospitals tend to be correlated. Finally, small countries like Denmark have a very limited number of hospitals and hospital sites.

Due to the above mentioned conditions, this study has merged the multiple hospital output vector in to at most two aggregated output measures to be able to estimate a "flexible" cost function in a small country context. This approach has the advantage that it maximizes the degrees of freedom for estimation and minimizes collinearity problems given the limited amount of observations.

The Danish DRG-system is the best available non-arbitrary output-measure in Denmark, which adjusts each output (discharge) for casemix and severity through the DRG cost attached to each discharge. Therefore, this study uses DRG-values to measure aggregated hospital output as in related Danish studies, e.g. Olesen, Jensen, & Svenning (2002); Olsen, Street, Svenning, Hvenegaard, & & Søgaard (2006); Olsen & Street (2007).

Outputs are defined as the total DRG-value or divided into inpatient and outputpatient production values per hospital site, depending on the degrees of freedom in each of the specified models.

The number of beds per hospital site is used as a proxy for the fixed cost. This approach makes it feasible to estimate the optimal number of beds (optimal capital stock) by application of a "flexible" functional form and the envelope condition (Preyra & Pink (2006a)[3]. Besides the number of beds per hospital sites, we have included cost shifters in the model. The limited number of degrees of freedom in the flexible functional forms together with potential multicollinearity did limit our ability to include very many of the potential cost shifters – e.g. patient characteristics. This study only includes a university dummy to adjust for the fact that University hospitals have higher costs.

Estimation technique

Two different estimation techniques have been applied to estimate the hospital cost functions (1). The translog, Cobb Douglas and quadratic forms have been estimated using conventional fixed effect panel data techniques which assume cost minimization. Besides, the Cobb Douglas model has been estimated by stochastic frontier analysis (SFA) which allows for technical inefficiency (Kumbhakar & Lovell (2000). A central difference from the FE-approach is that the disturbance term is decomposed in two terms.

The fixed effect regression approach was preferred to the SFA panel data models because it requires fewer assumptions. Fixed effect models allow for some unobserved heterogenity among hospitals and do not assume a distribution for an inefficiency term. Another advantage is that the fixed effect estimator is based on relatively few assumptions. It is not necessary to assume distributional assumptions on the individual effects or that individual effects are uncorrelated with the regressors. It is also difficult to test the above mentioned assumptions based on a "small country" sample.

The Hausman-test was conducted to confirm the choice of the FE-effect estimator versus the RE-effect estimator. Finally we interpreted the ability of the different outputs of the models to make reasonable economic inference – e.g. the sign in front of parameters.

Test of functional forms was not conducted since it does not make sense in a "small" country context, where few observations are available.

[3] It is not feasible to calculate a long run cost function from a standard Cobb Douglas cost function since it does not include squared terms.

The optimal relationship between size of hospital sites and activity

The optimal relationship between size of hospital sites and activity can be calculated from the short run cost function by application of the envelope condition, Varian (1884). Given the cost function (1) this calculation can be expressed as:

$$K^* = \frac{\partial g(C)}{\partial K} = 0 \quad (2)$$

Where the optimal size of a hospital site K^* is a function of the total output $Q_{T.}$ In case the estimated short run cost function is a translog cost function the optimal number of beds K^* becomes an exponential function of the total output $Q_{T.}$ Calculation of (2) for the quadratic form yields a linear relationship between activity Q_T and K^*[4].

The long run cost function

The long run cost function has been estimated in two different ways. The first approach uses the estimated short run cost function and the envelope condition to calculate the long run cost function. This means, that the first order condition of the short run cost function equal to zero defines the optimal relationship between beds and outputs as defined in (2). Substituting (2) in to the short run cost function yields the long run cost function. According to the conducted calculations in the enclosed appendix B this gives a second order equation for the long run translog cost function and the long run quadratic cost function respectively, Preyra & Pink (2006a).

The second approach applies an alternative way in which the cost function is estimated directly without use of the envelope condition (). The direct approach has been achieved by omitting the variable for the number of beds in the estimation of the cost function. The gained degrees of freedom are used to include two output measures instead of the total measure Q_T.

Economies of scale:

In agreement with Berger, Hanweck, & Humphrey (1987b) economies of scale are estimated in a way, which shows the relative rise in the costs, when it is accepted that the output is extended proportionally:

$$SE = \sum_j \partial \ln(C) / \partial \ln(Q_j) \quad (3)$$

[4] The formulas used to calculate optimal size of hospital sites in terms of number of beds and long run cost functions can be seen in the enclosed appendix.

Common for the scale expressions (3) is the fact, that SE-values less than 1 indicate economies of scale corresponding to cost increases, which are smaller than proportional with the output rises. Larger SE-values than 1 show diseconomies of scale. The latter means that a given output combination can be produced more cheaply at more small hospitals each with the same output-mixes rather than a big hospital. Scale expressions (3) is calculated according to Berger, Hanweck, & Humphrey (1987a) by using the average hospital for each of the defined size groups.

Data description

We analyse data for all Danish hospital sites except a handful of specialist sites. The data comes from The Danish Institute for Health Service Research (DSI) and The National Board of Health (2007b)[5]. For the operationalization of costs and outputs of the public hospital sites, panel data for adjusted operating costs, DRG values, and institutional characteristics of hospital in year 2004-2006 have been applied. The hospital sites are identified by their hospital number plus assigned additional digits, which identifies the individual sites[6]. Variables and descriptive statistics are shown in Table 2.1.

Table 2.1 shows that hospitals sites on average had adjusted operating costs in the range 530 to 616 million Danish Crowns. If one adds the production value for the two subgroups of activity (inpatient and outpatient) it can be seen, that the calculated total DRG-production value is averaging in the range 538 to 636 million. Comparing the reported adjusted operating cost and the DRG values show that the hospitals sites pay just above one Danish Crown per crown DRG value.

The adjusted operational costs and DRG values for the hospitals sites have been calculated for this study by aggregating the values of the individual patient's share of the adjusted operating costs and the DRG production value – firstly, to the ward level. Afterwards the values of the individual wards have been aggregated to give the total cost and DRG values for each hospital site.

There were different DRG classification systems in place for the period 2004 to 2006. Data from 2004 was grouped by the "2007 Grouper" and data for 2005 and 2006 was grouped by the "2008 Grouper". This means that data for 2004 is based on real 2007 prices and that data for 2005 and 2006 is based on real 2008 prices. Therefore the data from 2004 was adjusted for the inflation rate in 2007. Otherwise the casemix index would be contaminated with price effects where is should only be capturing differences in patient characteristics.

The included institutional characteristics are number of beds and a dummy for university status. The average number of beds per hospital site is in the range 265 to 281, but this average covers large

[5] It is an unbalanced dataset since the National Board of Health did not public data for all hospitals in 2006. Data is expected to be updated in august 2008.

[6] The aggregations of data from individual patient level to site level and definition of site codes have been conducted by customized SAS-programming at DSI due to external funding from the Danish VAT-fond.

variation between hospitals sites (e.g. min. 9, max.1136 in 2005). Furthermore table 2.1 shows that the average percentage of university hospitals sites was approximately 21% in the period 2004 to 2005. Finally, it should be noted that the data for 2006 is generally lower than the data in the previous year. This unexpected general trend is caused by missing data for 2006, which create bias since a relatively high part of the large sites are missing.

Atypical public hospital sites (psychiatric and special hospitals) are excluded from this study. This means that 6 hospital sites are excluded.

The Danish Institute for Health Service Research has gathered data on the number of average disposable beds per somatic hospital site from the hospitals. After summation of the figures for the single hospital sites figures are not always consistent with the overall official figures for the conglomerate hospitals published by The National Board of Health. The explanation is that the accumulated figures do not include psychiatric beds, which are included in the official figures for the conglomerate hospitals.

Table 2.1 Descriptive statistics for Danish public hospitals in the years 2004-2006.

Variable	Year	Description	Average	Std. dev.	Min	Max
		Dependent:	**in 1.000 crowns (Danish currency)**			
C	**2004**	**Adjusted operational costs**	**530.934**	**648.465**	**21.581**	**3.591.319**
	2005	**-**	**616.490**	**784.055**	**19.035**	**4.337.614**
	2006	**-**	**439.160**	**449.149**	**16.761**	**1.890.084**
		Independent:				
Q_1	**2004**	**DRG-value inpatient**	**328.514**	**379.452**	**0**	**2.123.694**
	2005	**-**	**361.908**	**477.284**	**0**	**2.516.725**
	2006	**-**	**232.824**	**231.748**	**3.265**	**898.218**
Q_2	**2004**	**DRG-value outpatients***	**209.637**	**270.439**	**4.909**	**1.180.949**
	2005	**-**	**274.984**	**335.966**	**2.514**	**1.572.589**
	2006	**-**	**226.344**	**247.555**	**3.555**	**1.000.748**
Q_T	**2004**	**Total DRG-value (Q_1+ Q_2)**	**538.152**	**592.734**	**22.968**	**3.304.644**
	2005	**-**	**636.892**	**788.952**	**12.098**	**4.089.314**
	2006	**-**	**459.168**	**468.839**	**15.163**	**1.898.966**
		Independent control variable:	**in no. of beds and percentage of hospitals**			
K	**2004**	**Average no. of staffed beds**	**281.6**	**250.9**	**25.6**	**1107.1**
	2005	**-**	**265.1**	**259.5**	**9**	**1136.7**
	2006	**-**	**176.0**	**152.1**	**9**	**517**
UH	**2004**	**University hospital**	**0.2105**	**0.4113**	**0**	**1**
	2005	**-**	**0.2182**	**0.4168**	**0**	**1**
	2006	**-**	**0.0968**	**0.3005**	**0**	**1**

*** Including the value of gray zone DRG activity.**

Source: Danish National Board of Health

The adjusted operating costs are the account of costs of the National Board of Health for the treatment of the somatic inpatients, who have been taken to a hospital according to The Danish National Patient Register. The adjusted operating costs are measured as the total operating costs minus costs that do not contribute to somatic inpatient treatment. The National Board of Health uses the hospital reports of the total operating costs, which are deducted the actual cost of psychiatric services, leasing and rent, spending for other patient treatments than the own patients of the hospital (due to free choice), laboratory services to general practitioners, the cost of medicine at the outpatient wards and adjustments for differences in accounting practice and unpaid services between hospitals.

This gives "the adjusted operational costs" as the Department of Health subsequent adjusts for the costs of outpatient treatment, and the cost for patients, who are hospitalised in particular long time and internally funded research to get "the adjusted operating expenses", Department of Health (2007a).

The DRG value is measured in local currency via the Danish casemix system called DK-DRG. The total value of DRG production for each hospital site is divided into two output categories. The two outputs are: a) the production value of inpatients (full time stationary patients) and b) outpatients including grey zone patients and emergency patients.

Outpatients consist of outpatient and part day visits, while inpatients are patients, who have been hospitalised at least 24 hours. Grey zone patients which in this study are treated as part of outpatient activity, are patients that the hospital staff both can chose to treat as outpatient as well as inpatient (in connection with hospitalization). These patients are settled with a grey zone DRG rate, calculated as the average between what it costs to perform the surgery or outpatient treatment, and the corresponding price by carrying out the same treatment in stationary prices. The idea is that hospitals are given a financial incentive to treat grey zone patients as outpatients.

The average number of disposable beds per hospital site is used as a proxy for the size of hospital sites and fixed inputs, while the dummy variable university hospital is used as control variable for hospital sites with University status.

3. Empirical results

Table 3.1 shows the results of the short run cost models in the Cobb Douglas, the translog and the quadratic model specification. For comparison purposes we have estimated both cross section and fixed effect panel data models for the Cobb Douglas model.

Tabel 3.1: Regression results - short run cost functions

	Cobb Douglas				Translog	Quadratic
	SFA	SFA	SFA	FE	FE	FE
Variable	2004	2005	2006	2004-2006	2004-2006	2004-2006
Intercept	-0.2315***	-0.1409***	-0.1717***	-0.1423*	-0,1966**	-0,0297
Inpatients (DRG value)	0.5853***	0.6016***	0.5402**	0.4425***	-	-
Outpatients (DRG value)	0.2852***	0.2684***	0.4162***	0.2736***	-	-
University hospital (dummy)	0.1364**	-0.0255	0.1189***	Not estimated	-	-
Total DRG value of in- and outpatients				-	0.6921***	1.4800***
Avg. no. of beds	0.0836	0.1265**	-0.0027***	0.0511	-0.0403	-1.2360**
Total DRG value^2				-	-0.0262	0.1266**
Avg. no. of beds^2				-	-0.0967	0.8131***
Interaction term				-	0.0938	-0.5377***
Number of observations	54	54	31	139	143	143
Number of hospital sites	54	54	31	54	60	60
R^2						
Within				0.6028	0.7717	0.7657
Between				0.9837	0.9778	0.9517
Overall				0.9763	0.9663	0.9374
F-test (5,78)				28.11***	105.55***	30.10***
Hausman chi2(5)						
Corr ($u_{i,X}$)				0.8389	0.8606	-0.9124

The stars show that the estimates are different from zero at the levels *** 1%, ** 5%, *** 10%.
Note: Estimated using stata version 9.0.

For the Cobb – Douglas and translog specification the beta estimates should be interpreted as elasticities while they in the quadratic form indicate the absolute increase in costs due to an increase in 1 unit of output. The Cobb-Douglas and the tranlog model show that elasticities for inpatients are higher than for outpatients and that the results are quite similar for cross section and panel data specification except for the outpatient elasticity being higher in 2006 than in 2004 and 2005. This deviation is probably due to missing data in 2006 as mentioned in the data description.

The university hospital dummy are positively significant for the 2004 and 2006 cross section analysis indicating that structural quality incurs higher cost. The beta estimate for the average number of beds changes sign and significance across the model specifications leaving the effect ambiguous. The regression results are used for two purposes. Firstly they are used to estimate the long run cost function based on the envelope condition which are showed together with the direct approach to long run cost function in table 3.2. Secondly they are used to estimate scale elasticities shown in table 3.3.

Table 3.2: Regression and calculated result - long run cost functions

	Translog & Envelope condition	Translog, FE (without beds)	Quadratic & Envelope condition
Intercept	-0.1950	-0.1709***	1.4980
Total DRG value	0.6845	-	2.8092
Total DRG value^2	-0.0043	-	0.3524
Inpatients DRG value	-	0.5388***	-
Outpatients DRG value	-	0.4100***	-
Inpatients DRG value^2	-	0.0879**	-
Outpatients DRG value^2	-	0.1094***	-
Inpatients*Outpatients	-	-0.1483***	-
R^2			
Within	-	0.6880	-
Between	-	0.9834	-
Overall	-	0.9764	-
F-test (5,75)	-	27.36***	-
Corr ($u_{i,X}$)	-	0.6297	-

The stars show that the estimates are different from zero at the levels ***: 1%,**:5%,***:10%.
Note: Calculated and estimated in stata version 9.0.

Table 3.2 shows the three long run cost functions. The first version of the translog function and the quadratic function are calculated from the short run cost functions in tabel 3.1 by substitution of (2). The second version of the translog model is a directly estimated fixed effect long run cost function. In this model the total output vector has been divided in to two output measures - inpatient and outpatient DRG value and no cost shifters have been included to avoid collinearity.

All models are 2´ order functions of the applied output vector. Again, the beta estimates of the translog cost functions should be interpreted as elasticities whereas the betas of the quadratic model show the absolute increases in costs.

Table 3.3 shows estimates of economies of scale for the alternative functional forms when applying the short run cost and long run cost functions respectively. All scale estimates are calculated for 4 size groups measured by the number of beds to obtain information on the shape of the cost curve. The size groups are defined by quartiles. The smallest size group (1.quartile) consists of the 25% of hospital sites, which has the smallest number of beds, while the other size groups called 2.quartile, 3.quartile and 4.quartile consist of hospitals with a size which lie between the quartiles[7]. All

[7] The quartile values used to construct the sub-groups of sizes: 1.quartile=50.9, 2. quartile= 229.0, 3.quartile= 356.6.

estimates are based on fixed effect models from table 3.2 and 3.3. For the translog and quadratic model the scale estimates within each group are calculated at the sup-group median.

Table 3.3: Scale estimates

	Short run			Long run		
Groups of hospital sites	Cobb Douglas	Translog	Quadratic	Translog (envelope condition)	Translog (directly estimated)	Quadratic (envelope condition)
All sites	0.7160	0.7086	1.0826	0.6493	0.8907	1.0545
1. quartile	0.7160	0.6235	1.3517	0.6897	0.6948	1.0138
2. quartile	0.7160	0.6688	1.3277	0.6796	0.8462	1.0927
3. quartile	0.7160	0.6972	1.2241	0.6716	0.9630	1.2239
4. quartile	0.7160	0.7206	0.9657	0.6674	1.0168	1.3068

The short run estimates in table 3.3 shows that results are dependent on the functional forms. The Cobb Douglas model and the translog model indicate significant economies of scale for all size groups and the quadratic form show constant or decreasing economies of scale.

The translog model expresses decreasing economies of scale and the quadratic model expresses decreasing diseconomies of scale as the size of sites are increased. Overall the models suggest that results of the flexible model depend on the applied functional form while the nested Cobb Douglas and the translog models yield similar results. However, a central difference is that the translog model allows estimation of non constant estimates. The results indicate that scale estimates are increasing with the size of the hospital sites which should be interpreted as economies of scale are declining as hospitals become larger.

The long run scale estimates in table 3.3 shows scale estimates based on (3) for the three alternative long run cost curves, taking into account that hospitals do not necessarily use the optimal capital in the short run and that hospitals can change the amount of capital according to the activity level in the long run. The LR results are similar to the short rum in the sense that the results are sensitive to the functional form. Besides the directly estimated translog model indicates that results are sensitive to the two alternative long run estimation approaches.

The two translog models indicate presence of scale economies while the quadratic form indicates constant or decreasing returns to scale. However, while the tranlog model based on the envelope condition finds that scale effects have a low variation between hospital size groups (0.69 for the largest hospitals and 0.67 for the smallest) the variation is larger for the direct approach (0.70 for the smallest to 1.02 for the largest hospitals).

The translog scale estimates for the largest size groups lie around 0.67 or very close to the value 1, equivalent to constant economies of scale in the long term. Thus, there is nothing in the translog models to indicate that the hospital sites experience diseconomies of scale in the long run, as the scale estimates are not significantly above the value 1.

Overall, we identify significant to moderate long run economies of scale, when applying two alternative translog cost function. However, using a quadratic functional form we identify constant economies of scale for the medium sized sub-groups and decreasing economies of scale for the largest sub-groups.

Figure 3.1: Long run economies of scale and size of hospital sites

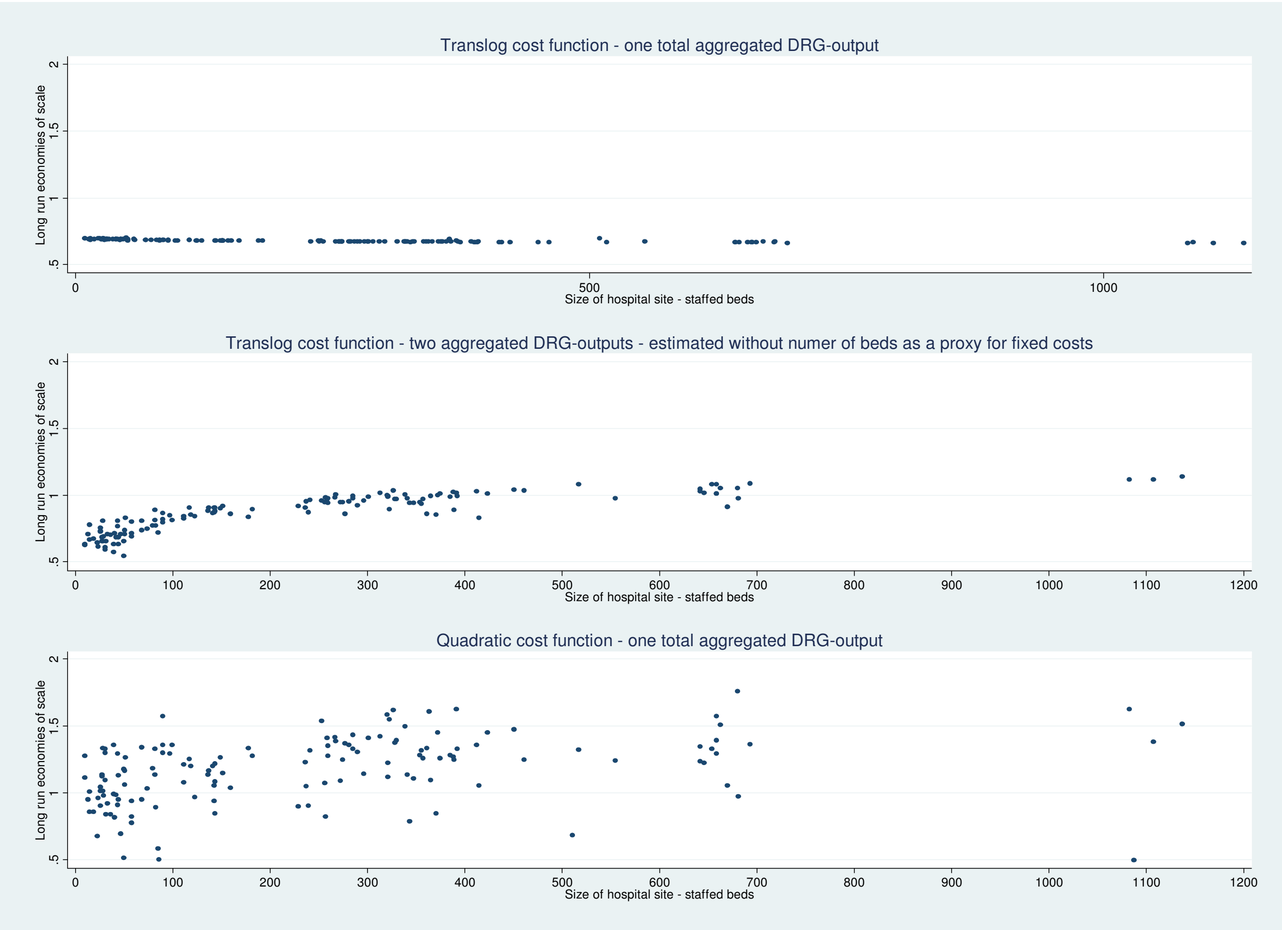


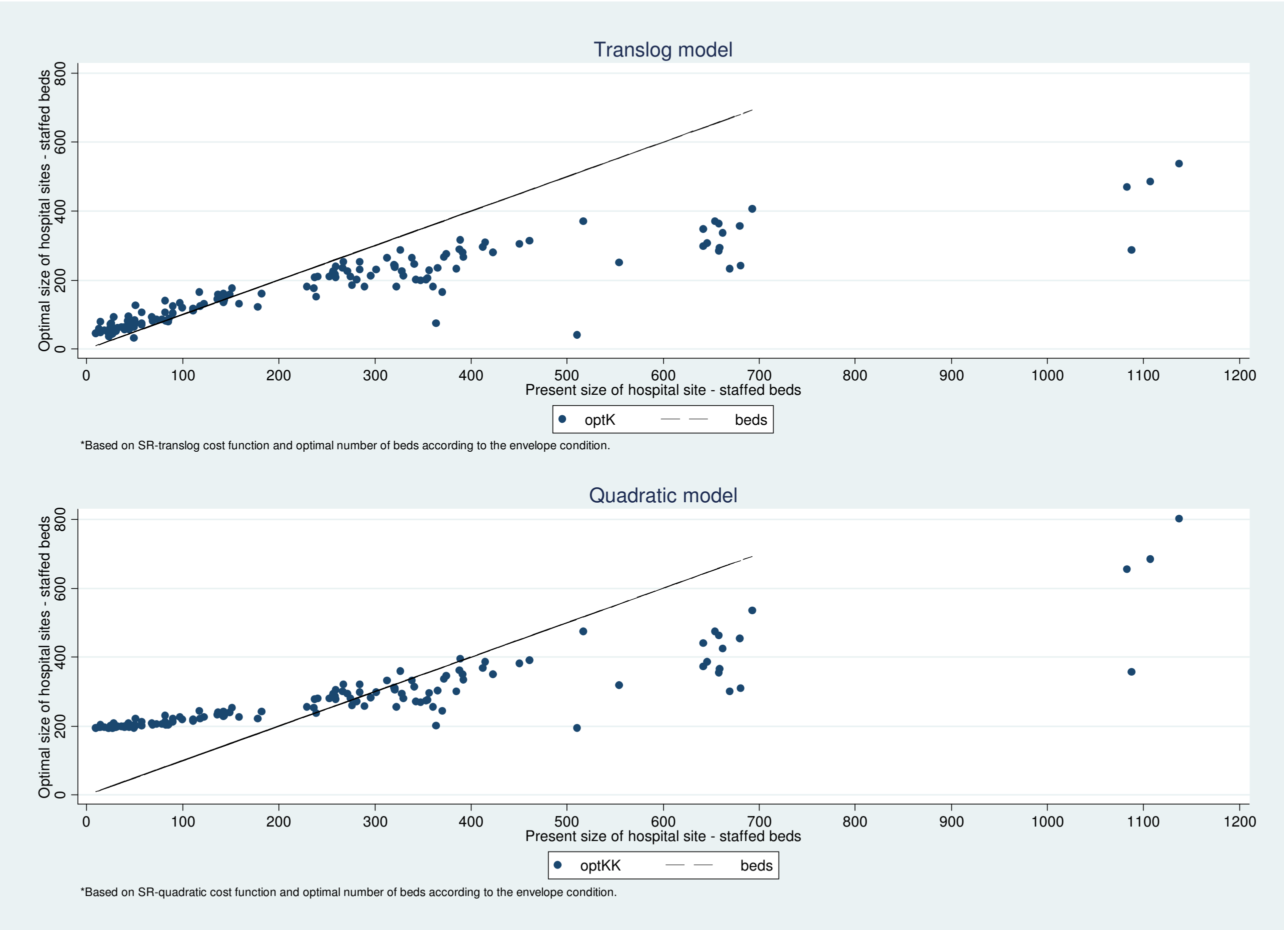
Figure 3.1 show the LR scale estimates as a function of hospital size for the three different LR model specifications. The figure shows that LR scale estimates for the translog model using the envelope condition lies below 1 for all hospital sizes whereas they start to exceeds 1 for hospitals above around 400 beds in the direct translog LR model. The estimates based on the quadratic form show less correlation between hospital size and LR scale estimates even though a positive trend can be detected with increasing size of hospital sites.

In contrast to table 3.3, which showed scale estimates for the median hospital sites data in each size groups (representative sites), figure 3.1 shows the short run scale characteristics for all observed hospital sites. The smallest quartile has estimates in the range of 9-50.9 beds, while the following three size groups (2.Quartile, 3.Quartile and 4. Quartile) have observations in the intervals 50.9-229.0, 229.0-356.6 and 356.6-1136.7 beds. Figure 3.2 also shows that there are three outlier observations. On closer examination, we have not taken the opportunity to exclude these observations. Finally, figure 3.1 shows that there are relatively few observations among the largest hospital sites. This makes it particularly important to review these units as they also can be regarded as extreme observations in the data sets.

The optimal relationship between size of hospital sites and activity (beds)

The estimation of the "optimal size" of hospital site based on (2) gave an optimal size of hospital sites of 204.9 beds for the median Danish hospital site in the translog model and 275.2 beds for the quadratic functional form.

Fig.3.2: Optimal sizes of site as a function of size – short run*



***The number of beds has been used to illustrate the size of hospital sites instead of the DRG values.**

In figure 3.2 the estimated "optimal size" of each site is shown as a function of present size. From the 45 degree line it is illustrated how the "optimal" size of each site deviates from the present site (45 degree line). The results of both models indicate that the small and medium sized sites with less than 204 or 275 beds are too small while the lager sites are too large. Both results are not far from the previous international literature that point to optimal sizes of hospital sites between. An example is a recent Canadian study which estimated an optimal value of 179, 5 beds, Preyra & Pink (2006).

4. Discussions

This study shows that parametric estimates of economies of scale and "optimal" size of hospital sites in Danish public hospital are sensitive to the chosen model specification despite the fact that the applied approaches are inspired by former studies of hospital cost functions.

The translog cost function, which has become a standard approach, has been applied in many studies of economies of scale characteristics in the hospital sector, Bamezai & Melnick (2006),Weaver & Deolalikar (2004), Scuffham, Devlin, & Jaforullah (1996) and in a number of studies of hospital efficiency, Rosko & Mutter (2007). In the same way, the Cobb Douglas function is a standard functional form, but is has been less used for the the estimation of economies of scale since the development of the more flexible functional form – e.g. the translog model, Chistensen, Jorgenson, & Lau (1973). The only functional form which is not very frequently used is the quadratic form even though it is a flexible functional form like the translog model (Preyra & Pink (2006b). This can be due to the fact that the coefficients are more difficult to interpret in the quadratic form. On the other hand the quadratic form has the advantages that it does not require logarithmic transformation which can result in a transformation problem regarding the dependent variable if heteroscedasticity is present in the model.

From one perspective, the translog and quadratic functional form features the great advantages that they are flexible forms, which minimize the risk of misspecification. From another perspective, increased flexibility requires an increasing number of parameters to be estimated. The last argument is a "small country" challenge. Firstly, the small number of observations means that only a small number of aggregate output measures can be applied in flexible functional form despite the fact that hospitals have multidimentional outputs. Secondly, flexible functional forms have a latent multicollinearity problem, which is likely to be more sever in a "small country" context, where the number of hospital is low, Scott & Parkin (1995), Aletras et al. (2007) and Kristensen (2008).

The correct structural model requires all parts of the flexible functional model including squared and interactions terms to be included, so that the model can capture varying economies of scale.

Unfortunately one cannot include more than one or two different output measures without getting multicollinearity since both the multiple output vector and cost shifters tend to correlated.

To try to minimize these obstacles the feasible approach of this study, has been to aggregate the output vector to one aggregated output and to use only two cost shifters. This is done to have enough degrees of freedom left for model estimation whit out getting more collinearity than the nature of the flexible forms imposes through squared and interactions terms. Besides, the unit of analysis has been defined as sites instead of conglomerate hospitals. Both to define the policy relevant unit of analysis, but also to increase the number of observations which are higher for hospital sites than conglomerate hospitals.

The above mentioned approach to aggregate to one output index can be discussed. From one point of view the multi dimensional output can only be aggregated into a lower dimension if the original dimensions are broad broad isoresource categories. From another point of view it can be argued that this is the case in DRG-systems.

Furthermore, the Cobb Douglas model has been applied to try to avoid multicollinearity. The lack of flexibility in the Cobb Douglas model implies that a fewer number of parameters have to be estimated than in the "flexible" cost function. From one point of view, this may be preferred in situations, where only a relatively small number of degrees of freedom are available as in the "small country" case. From another angel, use of the Cobb Douglas model implies constant scale elasticities which decreases flexibility an increases the risk of misspecification. The inflexibility of the Cobb Douglas model also shows that it can not be used to calculate optimal scale and optimal size as a function of size or activity as it can be seen form table 3.3 where the short run scale estimates become constant. The implication is that the Cobb Douglas only can be used as a supplementary measure as it was the case in this study.

More panel data could help to give more degrees of freedom and minimize multicollinearity through more variation in data, although it would not be able to solve the whole problem in case the output vector were broken down on homogeneous resource groups as all hospital specialties, wards or DRG-groups. The amount of extra parameters would increase relatively more than the degrees of freedom gained from extra years of panel data. Anyway we plan to update the data set with the latest data from 2006 to make it complete an balanced for the period 2004-2006.

In this paper we did not use more than three years of panel data. The argument is that changes in DRG groupers, DRG values and allocation of costs and other structural changes mean that it makes less sense to compare more years, Danish Regions, the Ministry of Finance, Health, and Ministry of the Interior and the Ministry of Health (2007).

Another consequence of the above mentioned "small country" problem is that the number of data is too small to conduct specifications tests to determine a "correct" functional form. Outliers will simply influence too much. Instead the results of several functional forms are presented to "measure" sensitivity to different functional forms.

-Discussion of the implications of the definition of the concept of cost. Fixed cost versus variable cost.

The Hausman test confirmed that the fixed effect model is preferred to the random effect model. This indicates that there is some kind of correlation between the individual effects and the covariates. Even though the fixed effect model was preferred it still has some disadvantages. For example that time invariant effects cannot be estimated. This prohibits relevant covariates to be include – e.g. teaching status, which is believed to be a structural cost driver in hospitals. In addition, optimally consistent estimators require that both time series and cross sectional dimensions go to infinity.

Maybe the old technology is not representative for the future technology in the new hospitals. The Danish regulators expect a one time productivity growth of 30%. This is indicating that the technology might be quite different after the rebuilding programme has been implemented.

Since the FE-approach is a version of regression analysis cost function estimation usually requires that it is assumed that hospitals apply variable input in a cost-minimizing way - i.e. that the hospitals are on the cost frontier (and that the hospitals cannot alter the number of beds in the short run). The model does not take into consideration the possibility of a technical inefficiency in the hospital production, which, however are documented in earlier studies of the productivity at both Danish hospitals, Olsen et al. (2006), Olsen & Street (2007) and current reports from the authorities, Danske Regioner (2007) and foreign studies, Farsi & Filippini (2006), Hillside & Rosenman (2001a),Rosko (2004), Rosko & Mutter (2007). On the other hand it has been claimed that scale estimates do get confounded by technical inefficency.

5. Conclusion

Overall this study shows significant to moderate long run economies of scale for all size groups of the Danish hospital sites in 2004-2006, when applying two alternative specifications of translog cost functions. These results indicate an L-shaped unit cost curve. However, using a quadratic form this study identifies constant economies of scale for the medium sized sub-groups and decreasing economies of scale for the largest sub-groups. This illustrates a U-shaped unit cost curve.

According to the scale results it may be appropriate to consolidate the production of the small hospitals on fewer units from a cost point of view. This may imply merging the small hospitals in the smallest size groups with hospitals in the medium size groups. In both cases taking into account the other determinants that may be important for whether it is appropriate to concentrate hospital

production - such as the need for a local emergency facility, the quality of hospital services, transport costs and opportunity costs through increased travel time.
Furthermore, the “optimal” number of bed per hospital site is estimated to be either 204 or 275 beds per site depending on whether the functional form is a translog or quadratic flexible functional form.

However, the investigation did not immediately give us the data to draw conclusions concerning the consolidation of hospitals, leading to hospital sizes with more than about 1200 beds, as it will lead to extrapolation in relation to the available data. It is in other words, not known whether the unit cost will decline (be L-shaped) or it will be U-shaped when hospital size increases above 1200 beds Overall, this study supports the hypothesis that there may be cost advantages (or no disadvantages) for the smallest sub-group in producing hospital services on larger hospitals in the Danish hospital system than it was the case in 2004-2006. However, there should not be taken health policy conclusions solely on the basis of this study, which is based on panel data only for years 2004-2006. The findings should be investigated further - for example through the involvement of different data and more model specifications, alternative methods such as data envelopment analysis (DEA), the measurement of the uncertainty of the estimates, and reports about the potential efficiency gains from consolidation.

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