Efficiency of Community Level Midwifery Care in Indonesia: a Stochastic Frontier Analysis

by

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1 Introduction

In 1989, the Indonesian government introduced a village-level midwifery service that was made available throughout the entire country. Its purpose was to improve public access to skilled attendants in order to reduce maternal, childbirth and infant mortalities. The state-employed midwives act as the providers of antenatal and postnatal care, as well as home-based delivery care (averaging approximately 36 deliveries per year per midwife). In addition, midwives provide other non-maternal health services such as family planning, immunisation and child health care. Importantly, all midwifery services are provided free of charge to those classified as poor.

Village midwives undertake one year of diploma-level training (as opposed to three years of degree-level training required of hospital midwives) supplemented afterwards with specialised in-service training courses. Their service is directed towards the residents of their assigned village, although some midwives can be assigned duties that cover more than one village. On average, each midwife provides care to around 3,500 people. In addition, for a few hours per week midwives must provide care to pregnant women and children outside of their assigned villages at sub-district health centres. See Makowiecka e*t al* [12] for further details.

Midwives are employed as civil servants, holding either a permanent position that pays, on average, an annual salary of US$1,768 (here Indonesian Rupiah have been converted to US dollars using the annualised exchange rate for 2006: US$1 = IDR9200), or limited term contracts. The latter includes central government contracts, financed by the Ministry of Health, that pays on average US$1,164 that are more lucrative than the average US$866 per annum paid under district government contracts. In addition to salary, midwives receive income from fees charged for their service, where this income can be sourced either directly from the user, or, if the user is classified as poor, indirectly from the government. See Ensor e*t al* [6] for further details.

The midwife can be viewed as a maternal health care unit (or DMU=decision making unit, the terminology used by Jacobs e*t al* [10] in health contexts), operating at the community level using resources provided by the government (e.g. salaries and fees for serving the poor, pharmaceutical drugs) as well as their own resources and inputs (e.g. pharmaceutical drugs and supplies, and owned/rented consulting facilities). Our study of levels and determinants of cost efficiency is designed to provide policy makers with insights into the cost effectiveness of the village midwifery programme. As the midwifery service is by not run along profit maximisation lines we do not *a priori* expect to identify midwives operating near to the cost efficiency frontier.

The econometric methodology we apply to study the determinants of cost efficiency of midwives is the dependent errors Stochastic Frontier Model (SFM) developed in Smith [18]. The dependent errors SFM represents a statistical generalisation of the popularly applied Standard SFM (see, for example, the major texts Coelli *et al* [5] and Kumbhakar and Lovell [11]) in that the model error components need not be treated as statistically independent.

2 Data & Descriptive Statistics

As part of a complex evaluation of the Village Midwife programme undertaken by the IMMPACT Project (see Achadi e*t al* [1]), two districts, Serang and Pandeglang of Banten Province in West Java were selected for detailed analyses. For studying costings, a stratified random sample of 207 midwives (out of a total of 737 midwives) providing services in 216 villages (out of a total of 711 villages) were obtained. Stratification was based on the type of work assignment and the distance based on midwife's main place of residence (the village assigned) to the nearest district hospital in kilometres. The former were observed in three categories: (i) a midwife resides in a village and has sole responsibility for that village alone; (ii) a midwife responsible for a particular village does not reside in that village; and (iii) a midwife is responsible for more than one village.

An ingredients approach was used to identify the costs associated with the resources and inputs used either directly or indirectly for village-level midwifery care, as well as those used at different service delivery points; for example, at the expectant mother's home, and at the outreach facilities in the village. These inputs and resources include personnel time (the midwives time and also the time of other community level workers if they were found to be assisting midwives), travelling time (travel to different service delivery points/outreach facilities as well as to the health facilities they are administratively assigned to), pharmaceutical drugs and health supplies, use of utilities, and capital and other equipment. Other costing factors include the remoteness of villages, the number of assigned villages, whether the midwife is resident in the village, and the type of contract held by the midwife. Data on costs was sought from midwives using two questionnaires, one to collect costing data, and another to collect time-use information. The costing questionnaire took place during August–October 2005 and collected information on the resources and inputs used by a midwife at different service delivery points in the village(s) they were assigned to serve. Secondary information like services output over a longer period (e.g. for the whole year), both at midwives/village level and health centre level were also obtained from health centre records. In the second questionnaire, the midwife reported their allocation of time to maternal and non maternal health services at different service delivery points. These data were then used to apportion time to costs of the resources and inputs used for maternal and non maternal health services. See Quayyum e*t al* [15] for further details.

Table 1 is a cross tabulation of average cost by the stratification components: work assignment and distance. It shows that variation in costs are indeed due, at least in part, to these components, and so our cost model must control for these factors.

This study in estimating the frontier costs function assumed that the level of yearly total costs (*tcusd*- in US dollars converted using the exchange rate at 2006 – 1 US- IRs 9200) of midwifery care at the village level would be determined by the amount of total salary (*tsal*) of midwives, the total amount of rent for facilities (*rent2*), number of delivery care by the midwife during the year (*bdndel*), the type of midwives (purely village midwives – *bdesa*=1 as against health centre midwives/midwives co-ordinator 0), the type of village they are serving (*urban*=1 as against village in rural and/or remote areas 0.), the villages either located in Serang district (which is more developed - denoted by a dummy in the model *seryes*=1) and more urban as against the villages located in Pandeglang district, the distance (in km) of the village from the nearest health centre (*dis_hc*), type of assignment and residency: responsible for one village and resident there (*resp1res*=1) or responsible for one village but not resident there (*resp1notr*=1) as against those who are responsible for more than one village (control group), number of villages covered by the midwives (*num_ vil*), the number of outreach facilities covered (*cp_ rpos3*) and proportion of poor in the assigned village (*num_ poor*). Other provider like traditional birth attendant (*TBA*) was expected to effect the level of midwives' maternal and child health activities and hence the total number of TBA (*num_ tba*) was considered as a determinants of the total cost of midwifery care. *Non_ mch* the total time for all the non-maternal health services provided by the midwives. Table 2 lists sample statistics of these variables.

3 Econometric Methodology

3.1 Model Construction

The dependent error SFM is given by

$$\log Y = x'\beta + V + U \tag{1}$$

where cost Y is positive-valued, x $(k \times 1)$ is a vector of regressors (i.e. known functions of the inputs) assumed exogenous, and β $(k \times 1)$ is a vector of unknown parameters. The cost inefficiency error component $U = u > 0$ is a random variable with distribution function (cdf hereafter) $F(u) = \Pr(U \leq u)$ that is assumed continuous, independent of x, but dependent possibly on unknown parameters that are collected into a vector δ_u. Likewise, the noise error component $V = v \in I\!R$ has cdf $G(v) = \Pr(V \leq v)$ that is assumed to be continuous, independent of x, but dependent possibly on unknown parameters collected in vector δ_v. By Sklar's representation theorem (see Sklar [16] and [17], as well as Nelsen [13]) the joint cdf of U and V is given by

$$\begin{aligned} H(u,v) &= \Pr(U \leq u, V \leq v) \\ &= C_\theta(F(u), G(v)) \end{aligned} \tag{2}$$

where $C_\theta(\cdot,\cdot)$ is a family of bivariate copulas that is indexed by a vector θ that consists of unknown, estimable dependence parameters. For example, the Plackett family of bivariate copulas that emerges as preferred in Section 4 below is given by

$$C_\theta(a,b) = \frac{1}{2(\theta-1)}(s-t) \tag{3}$$

where, for $0 \leq a, b \leq 1$, the functions $s = 1 + (\theta - 1)(a + b)$ and $t = \sqrt{s^2 - 4ab\theta(\theta-1)}$, and scalar parameter $\theta > 0$. Other examples of families of bivariate copulas indexed by a scalar parameter θ appear in Table 3. Finally, it is assumed that θ has no elements in common with any in $(\beta, \delta_u, \delta_v)$.

The standard SFM arises when the family $C_\theta(a,b)$ consists of the single member corresponding to the Product copula $\Pi = ab$; that is, when U and V are independent (in which case the standard SFM presented in texts such as Coelli e*t al* [5] and Kumbhakar and Lovell [11] follow). The SFM specification introduced via (2) is a statistical generalisation of the standard SFM model provided that it includes the standard SFM as a special case, enabling formal testing of the dependent error SFM over the standard SFM. This applies in the case of the Plackett family of copulas (3) for it nests Π as the limiting case corresponding to $\theta \to 1$, so that the test of interest has as the null hypothesis $\theta = 1$.

The log-likelihood function given in the following section is derived from the distribution of the composite error $W = V + U$, where the random variable $W = w \in I\!R$ is continuous due to the assumptions on U and V. Let $h_\theta(w)$ denote the probability density function (pdf hereafter) of W, which can be derived from the joint pdf of $(U,V) = (u,v)$:

$$\begin{aligned} h(u,v) &= \frac{\partial^2}{\partial u \partial v} H(u,v) \\ &= f(u)g(v)c_\theta(F(u),G(v)) \end{aligned}$$

where $f(u) = \partial F(u)/\partial u$ and $g(v) = \partial G(v)/\partial v$ denote, respectively, the pdf of U and the pdf of V, and c_θ is the copula density of C_θ. The latter, for example, in the case of the Plackett family of copulas (3), is given by $c_\theta(a,b) = \theta\,(s - 2ab(\theta-1))\,t^{-3}$. Note that the dependence between U and V is captured entirely by the density weighting function $c_\theta(F(u),G(v))$. Of course, if U and V are independent then $c_\theta(\cdot,\cdot) = 1$ implying that $h(u,v) = f(u)g(v)$. Transforming $(U,V) \to (U,W)$ yields, as the pdf of $(U,W) = (u,w)$:

$$h(u,w) = f(u)g(w-u)c_\theta(F(u),G(w-u)).$$

Thus, the pdf of W is given by

$$\begin{aligned} h_\theta(w) &= \int_0^\infty h(u,w)du \\ &= E_U[g(w-U)c_\theta(F(U),G(w-U)] \end{aligned} \tag{4}$$

where $E_U[\cdot]$ denotes statistical expectation with respect to the distribution of U.

3.2 Log-Likelihood Function and Estimation

For a cross-section of n midwives assumed mutually statistically independent the log-likelihood is obtained directly from the pdf of W (4) as

$$\log L(\beta, \delta_u, \delta_v, \theta) = \sum_{i=1}^{n} \log h_\theta(\log y_i - x_i'\beta)$$

where y_i is the cost associated with the ith midwife, and x_i the corresponding regressor vector. Rather than apply maximum likelihood techniques (ML) directly to $\log L$, Burns [4] adapted Greene's [8] simulation estimator arguing that estimation based on maximising the simulated log-likelihood (MSL) is attractive in the dependent errors SFM because it will be rare that a closed form solution will exist for h_θ, likewise for the log-likelihood (for an exception see Smith [18, section 4.1]). Using the expectational form (4) for h_θ finds the simulated log-likelihood given by

$$\log \widehat{L}(\beta, \delta_u, \delta_v, \theta) = \sum_{i=1}^{n} \log \left(\frac{1}{Q} \sum_{q=1}^{Q} g(\log y_i - x_i'\beta - u_{iq}) c_\theta(F(u_{iq}), G(\log y_i - x_i'\beta - u_{iq}) \right)$$

where, for each observation $i = 1, ..., n$, there are Q random drawings taken from the distribution specified for U to construct $\{u_{iq}\}$. Provided Q is permitted to increase with n then the simulated log-likelihood $\log \widehat{L}$ is a consistent estimator of the log-likelihood $\log L$, and so too are the MSL estimates formed by maximising $\log \widehat{L}$ consistent for the model parameters $(\beta, \delta_u, \delta_v, \theta)$; for the properties of the MSL estimator see, for example, Gourieroux and Monfort [7]. For our application, sample size $n = 201$ and we choose to base our MSL estimates on $Q = 200$ drawings per observation.

Finally, we adopt a further improvement recommended by Burns that is designed to improve computational efficiency, namely, using quasi-random Halton sequences applied to the $(0, 1)$ unit interval as the basis for generating $\{u_{iq}\}$; see also Train [19] and Greene [8]. In particular, we generate the 4th iteration sequence using base 17, yielding a total of 83520 Halton numbers from which we discard all but $nQ = 40200$, these are then partitioned into 201 sets of 200 numbers. These sets of numbers are used repeatedly in each iteration of optimisation of $\log \widehat{L}$, where we select BFGS as our optimising algorithm. To form the estimate of the variance-covariance matrix of the MSL estimate we use the final iterate of the approximation to the inverse Hessian that is generated at each step of the BFGS algorithm.

4 Results

Stochastic frontier models (SFM) were estimated assuming Cobb-Douglas cost functions, from these midwife-specific cost efficiency estimates were constructed. Assuming a Normal - Half-Normal specification, we contrast the performance of two models in particular that are differentiated by assumptions placed on the error terms: the more general model – the Plackett SFM – permits dependence across error components, whereas this feature is not permitted in the standard SFM.

Our overall estimation results favour the Plackett SFM (maximised log-likelihood -54.60) over the standard SFM (maximised log-likelihood -63.49), this is also borne out by the Wald test on the Spearman rho coefficient (estimate -0.87 is significantly negative-valued). Turning next to the estimated cost function in our preferred Plackett SFM, the signs of the estimated coefficients of the following covariates - level of salary, rent of facilities, number of delivery care, villages being located in Serang district, midwives with the responsibility of only one village, total time spent on non-maternal and child care - suggests via a standard marginal analysis that as each increases, the costs associated with providing midwifery care increase. On the other hand, the more villages covered by a midwife lowers the cost of midwifery care.

Salaries constitute more than 50% of the total recurrent costs and significantly explains the level of total costs. The rent of the facilities and buildings constitute a major proportion (more than 50%) of the capital costs and affect the total costs (Quayyum e*t al* [15]). The delivery care, which are provided at the community level, either at the mothers' home, or health centre or at midwives private practice, are resource incentive both in terms of time and inputs compared to other services the midwives provide and hence have positive effects on the total costs of midwifery services. The cost of inputs like rental and building villages located in urban areas and in Serang districts are higher and positively affecting the costs. Besides, most of the midwives in these villages have been in the services for longer period and receiving higher salary. Midwives who are assigned with one villages are likely to have higher overhead costs than those who are assigned with larger number of villages, as result they have higher level of cost. The midwives who cover more than one village can distribute her overhead among the assigned villages and enjoys benefit of the economies of scale. However it should it should noted here that it is mostly the remote areas where the midwives covers more than one village, and it is in these area where the uptake of skilled care for delivery services are low, and number of poor population is higher. The average number of delivery in year in these villages is 26 as against 45 in villages where the midwives assigned with one village and resident there (Quayyum e*t al* [15]). So, increasing the number of villages to be covered by midwives is not necessarily a good idea. In fact estimates of average cost per women of reproductive age group in a villages suggest that there is not a wide variations, and it is important to determine whether the delivery care are provided largely to the poor population as studies suggest that even though the village midwifery programme contributed to the increased used of skilled care and professional attendance at delivery (Hatt e*t al* [9]), there are still inequality in use of skilled care among different socio-economic groups and certain regions of the country (Quayyum e*t al* [14], Achadi, e*t al* [2]).

We measure cost efficiency CE_θ following Battese and Coelli [3] via the conditional expectation:

$$
\begin{aligned}
CE_\theta &= E[\exp(-U)|W = w] \\
&= \frac{1}{h_\theta(w)} \int_0^\infty \exp(-u) h(u, w) du \\
&= \frac{E_U[\exp(-U) g(w-U) c_\theta(F(U), G(w-U)]}{E_U[g(w-U) c_\theta(F(U), G(w-U)]} \qquad (5)
\end{aligned}
$$

where (4) has been used to obtain CE_θ as the ratio (5). As $U > 0$, then $CE_\theta \in (0, 1)$, where values of CE_θ closer to unity are indicative of cost efficiency. CE_θ is estimated for each midwife by evaluating it at $\widehat{w}$, the vector of observed residuals. This procedure enables midwives to be individually ranked. Despite the clear preference for these data for the Plackett SFM over the Standard SFM, the comparison of estimated ranks across the two fitted models finds close agreement with Spearman rank correlation coefficient 0.973. However, clear differences emerge in the values themselves, with estimates of the preferred Plackett SFM distant from the cost frontier ($\max_i CE_{\theta i} = 0.81$), whereas that is not the case for estimates from the Standard SFM ($\max_i \widehat{CE}_{\theta i} = 0.95$). This can be seen in Figure 1 that plots both kernel-smooth cost efficiency distributions. This result is not wholly unexpected, because midwives do not behave according to profit maximisation principles, often being engaged in health promotion activities that do not directly contribute to their maternal health output. The fact that the better-fitting Plackett SFM is able to detect this vindicates our decision to use generalised SFM methodology.

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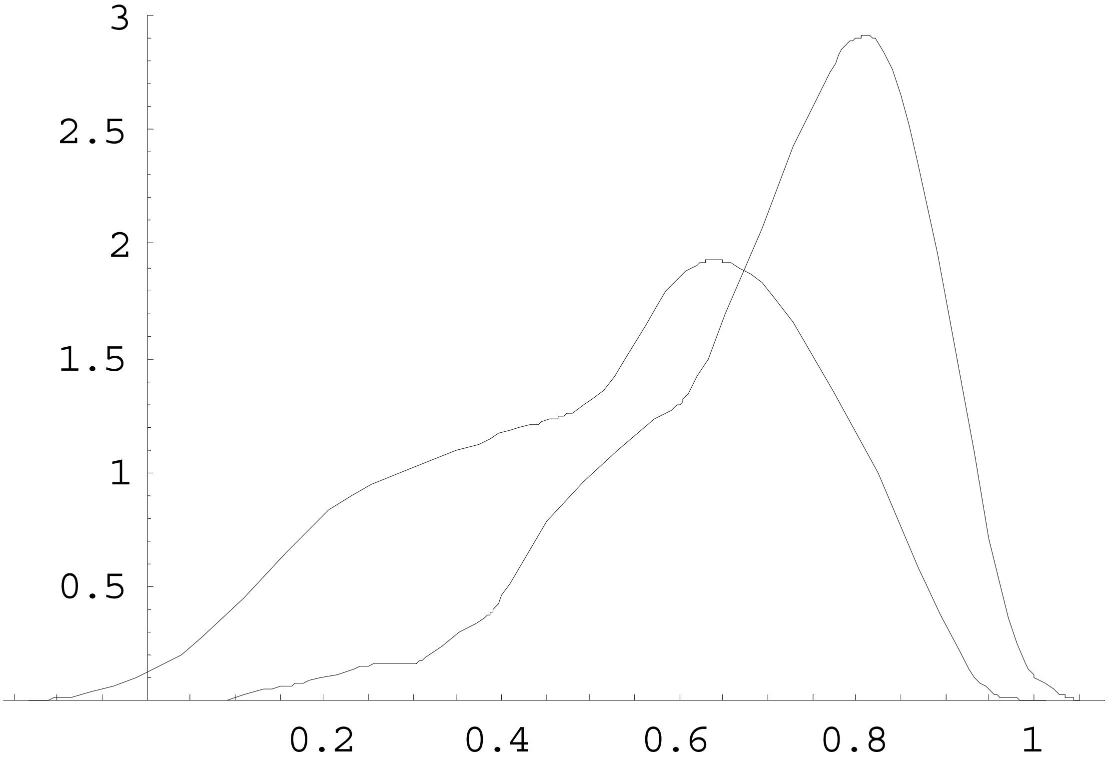

Figure 1: Kernel-smooth midwife cost efficiency distributions

Table 1: Examples of Bivariate Copula Families

Characteristics	obs	Mean (US$)	Std. Dev. (US$)	Min	Max
Distance to district hospital					
Nearest	68	4612.12	3689.14	1160.50	24052.72
Middle	64	4135.12	3004.02	984.72	20293.65
Remote	71	3541.52	2921.10	808.28	19221.14
Type of responsibility					
1 village + resident	66	5000.51	4259.06	1107.70	24052.72
1 village + non-resident	86	4060.02	2420.35	997.27	14749.54
More than 1 village	51	2951.80	2529.20	808.28	11666.40
Type of provider					
Village Midwife	160	3945.74	3059.83	984.71	24052.72
Health Centre MW/Coordinator	43	4614.74	3823.16	808.28	20293.65
Total	203	4087.38	3237.86		

Notes: (a) Source Quayyam e*t al* [15].

(b) Indonesian Rupiah converted to US dollars using the 2006 annualised exchange rate: US$1 = IDR9200.

Table 2: Summary Statistics

Variables	Mean	Std. Dev.	Min	Max
	8.095	0.630	6.695	10.088
ltsal	7.135	0.322	5.564	7.700
lrnt2	6.031	0.833	2.751	8.100
bndel	31.604	26.352	0	138
bdesa	0.791	0.408	0	1
urban2	0.114	0.319	0	1
seryes	0.527	0.500	0	1
dis_hc	6.087	5.392	0	41
resp1res	0.184	0.389	0	1
resp1notr	0.214	0.411	0	1
num_vil	2.274	1.480	1	10
num_poor	0.309	0.161	0.061	1.213
cp_rpos3	4.502	2.289	0	12
num_tba	4.040	2.594	0	15
non_mch	1.716	1.602	0	11.055

Table 3: Examples of Bivariate Copula Families

AMH [a]	
Copula C_θ [b]	$\frac{ab}{1-\theta(1-a)(1-b)}$
Density c_θ	$[1+\theta\,(ab+a+b-2+\theta\,(1-a)\,(1-b))]\,[1-\theta\,(1-a)\,(1-b)]^{-3}$
Parameter θ	$-1 \le \theta \le 1$
Spearman's S_ρ [c]	$\frac{12(1+\theta)}{\theta^2}\,\text{dilog}(1-\theta) - \frac{24(1-\theta)}{\theta^2}\log(1-\theta) - \frac{3(\theta+12)}{\theta}$
FGM [a]	
Copula C_θ	$ab(1+\theta(1-a)(1-b))$
Density c_θ	$1+\theta(1-2a)(1-2b)$
Parameter θ	$-1 \le \theta \le 1$
Spearman's S_ρ	$\frac{\theta}{3}$
Frank	
Copula C_θ	$-\theta^{-1}\log\left(1+\frac{(e^{-\theta a}-1)(e^{-\theta b}-1)}{e^{-\theta}-1}\right)$
Density c_θ	$\theta\left(1-e^{-\theta}\right)e^{-\theta(a+b)}\left[1-e^{-\theta}-(e^{-\theta a}-1)(e^{-\theta b}-1)\right]^{-2}$
Parameter θ	$-\infty < \theta < \infty$
Spearman's S_ρ [d]	$1-\frac{12}{\theta}(D_1(\theta)-D_2(\theta))$
Plackett	
Copula C_θ [e]	$\frac{1}{2(\theta-1)}\,(s-t)$
Density c_θ	$\theta\,(s-2ab(\theta-1))\,t^{-3}$
Parameter θ	$\theta > 0$
Spearman's S_ρ	$\frac{\theta+1}{\theta-1} - \frac{2\theta}{(\theta-1)^2}\log\theta$
Normal	
Copula C_θ	$\Phi_2(\Phi^{-1}(a), \Phi^{-1}(b); \theta)$
Density c_θ	$\frac{\phi_2(\Phi^{-1}(a),\Phi^{-1}(b);\theta)}{\phi(\Phi^{-1}(a))\ \phi(\Phi^{-1}(b))}$
Parameter θ	$-1 \le \theta \le 1$
Spearman's S_ρ	$\frac{6}{\pi}\arcsin\left(\frac{\theta}{2}\right)$

Notes: (a) AMH denotes Ali-Mikhail-Haq. FGM denotes Farlie-Gumbel-Morgenstern.

(b) For all copulas a and b are such that $0 \le a, b \le 1$.

(c) $\text{dilog}(z) = \int_1^z \log(t)\,(1-t)^{-1}\,dt$ is the dilogarithm function.

(d) the Debye function $D_k(z) = kz^{-k}\int_0^z t^k\left(e^t-1\right)^{-1}dt$, for k any positive integer.

(e) $s = 1+(\theta-1)(a+b)$ and $t = \sqrt{s^2-4ab\theta(\theta-1)}$.

Table 4: Cost Function Estimates

	Normal V - Half-Normal U	
	Product copula	Plackett copula
constant	3.040 (4.93)	3.286 (6.36)
ltsal	0.413 (5.09)	0.337 (5.01)
lrnt2	0.272 (7.52)	0.241 (8.95)
bndel	0.004 (4.23)	0.005 (6.23)
bdesa	0.014 (0.23)	0.043 (0.86)
urban2	0.197 (2.53)	0.170 (2.75)
seryes	0.203 (4.09)	0.193 (4.49)
dis_hc	0.003 (0.68)	0.001 (0.21)
resp1res	0.218 (2.76)	0.177 (2.28)
resp1notr	0.151 (2.00)	0.137 (2.23)
num_vil	−0.128 (−5.59)	−0.147 (−6.95)
num_poor	−0.125 (−0.83)	−0.032 (−0.32)
cp_rpos3	−0.009 (−0.77)	0.002 (0.19)
num_tba	0.002 (0.25)	0.002 (0.33)
non_mch	0.041 (2.57)	0.060 (3.90)
σ_u	0.495 (11.13)	0.991 (4.79)
σ_v	0.173 (6.42)	0.345 (2.61)
θ		0.021 (1.15)
$\log L$	−63.487	−54.498
S_ρ		−0.872 (−11.07)
λ [(b)]	2.858 (4.50)	2.874 (5.07)
σ^2 [(b)]	0.275 (7.07)	1.101 (2.21)
γ [(b)]	0.891 (20.64)	0.892 (23.47)

Notes: (a) Figures shown to 3 dp. t-ratio in parentheses.

(b) $\lambda = \sigma_u/\sigma_v$, $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\gamma = \sigma_u^2/\left(\sigma_u^2 + \sigma_v^2\right)$.